Mathematics-8

1 Rational Numbers

Exercise 1.1

1. Express the following rational numbers in standard form :

(a) $\frac{-56}{104}$

2	56
2	28
2	14
7	7
	1

2	104
2	52
2	26
13	13
	1

$\because$ HCF of 56 and 104

$= 2\times 2\times 2 = 8$

$\therefore \quad \frac{-56}{104} = \frac{-56\div 8}{104\div 8} = \frac{-7}{13}$

(b) $\frac{65}{-156}$

5	65
13	13
	1

2	156
2	78
3	39
13	13
	1

$\because$ HCF of 65 and 156 = 13

$\therefore \quad \frac{65}{-156} = \frac{65\div(-13)}{-156\div(-13)} = \frac{-5}{12}$

(c) $\frac{-84}{-91}$

2	84
2	42
3	21
7	7
	1

7	91
13	13
	1

$\because$ HCF of 84 and 91 = 7

$\therefore \quad \frac{-84}{-91} = \frac{-84\div(-7)}{-92\div(-7)} = \frac{12}{13}$

(d) $\frac{65}{125}$

5	65
13	13
	1

5	125
5	25
5	5
	1

$\because$ HCF of 65 and 125 = 5

$\therefore \quad \frac{65}{125} = \frac{65\div 5}{125\div 5} = \frac{13}{25}$

2. Fill in the blanks with >, < or = Sign :

(a) $\frac{-4}{7} \square \frac{-5}{13}$

$\because \quad \frac{-4}{7} = \frac{-4\times 13}{7\times 13} = \frac{-52}{91}$

(b) $-8 \square \frac{-15}{6}$

$\because \quad -8 = -8\times\frac{6}{6} = \frac{-48}{6}$

And, $\frac{-5}{13}=\frac{-5\times 7}{13\times 7}=\frac{-35}{91}$

$\therefore \quad \frac{-52}{91}<\frac{-35}{91}$

So, $\frac{-4}{7}<\frac{-5}{13}$

And, $\frac{-15}{6}=\frac{-15}{6}$

$\therefore \quad \frac{-48}{6}<\frac{-15}{6}$

So, $-8<\frac{-15}{6}$

(c) $\frac{-3}{8} \; \square \; \frac{6}{-11}$

$\because \quad \frac{-3}{8}=\frac{-3\times 11}{8\times 11}=\frac{-33}{88}$

And, $\frac{6}{-11}=\frac{6\times(-8)}{-11\times(-8)}=\frac{-66}{88}$

$\therefore \quad \frac{-33}{88}>\frac{-66}{88}$

So, $\frac{-3}{8}>\frac{6}{-11}$

(d) $\frac{-4}{13} \; \square \; \frac{9}{10}$

$\because \quad \frac{-4}{13}=\frac{-4\times 10}{13\times 10}=\frac{-40}{130}$

And, $\frac{9}{10}=\frac{9\times 13}{10\times 13}=\frac{117}{130}$

$\because \quad \frac{-40}{130}<\frac{117}{130}$

So, $\frac{-4}{13}<\frac{9}{10}$

3. Arrange in ascending order :

(a) $\frac{-4}{7}, \frac{5}{-42}, \frac{-3}{28}, \frac{-2}{21}$

$\because$ LCM of 7, 42, 28 and 21 is $(2\times 2\times 3\times 7)=84$.

2	7, 42, 28, 21
2	7, 21, 14, 21
3	7, 21, 7, 21
7	7, 7, 7, 7
	1, 1, 1, 1

$\therefore \quad \frac{-4}{7}=\frac{-4\times 12}{7\times 12}=\frac{-48}{84}$; $\frac{5}{-42}=\frac{5\times(-2)}{-42\times(-2)}=\frac{-10}{84}$;

$\frac{-3}{28}=\frac{-3\times 3}{28\times 3}=\frac{-9}{84}$; and $\frac{-2}{21}=\frac{-2\times 4}{21\times 4}=\frac{-8}{84}$;

Since, $-48<-10<-9<-8$

$\therefore \quad \frac{-48}{84}<\frac{-10}{84}<\frac{-9}{84}<\frac{-8}{84}$.

Hence, the ascending order is $\left(\frac{-4}{7}<\frac{5}{-42}<\frac{-3}{28}<\frac{-2}{21}\right)$.

(b) $\frac{-1}{4}, \frac{5}{-12}, \frac{-7}{24}, \frac{-9}{8}$

$\because$ LCM of 4, 12, 24 and 8 is $(2\times 2\times 2\times 3)=24$.

2	4, 12, 24, 8
2	2, 6, 12, 4
2	1, 3, 6, 2
3	1, 3, 3, 1
	1, 1, 1, 1

$\therefore \quad \frac{-1}{4}=\frac{-1\times 6}{4\times 6}\Rightarrow\frac{-6}{24}$; $\frac{5}{-12}=\frac{5\times(-2)}{-12\times(-2)}\Rightarrow\frac{-10}{24}$;

$\frac{-7}{24}=\frac{-7}{24}$; and $\frac{-9}{8}=\frac{-9\times 3}{8\times 3}\Rightarrow\frac{-27}{24}$;

Since, $-27<-10<-7<-6$

$\therefore \quad \frac{-27}{24}<\frac{-10}{24}<\frac{-7}{24}<\frac{-6}{24}$;

Hence, the ascending order is $\left(\frac{-9}{8}<\frac{5}{-12}<\frac{-7}{24}<\frac{-1}{4}\right)$.

(c) $\frac{-5}{6}, \frac{10}{3}, \frac{-7}{-2}, \frac{13}{8}$

2	6, 3, 2, 8
2	3, 3, 1, 4
2	3, 3, 1, 2
3	3, 3, 1, 1
	1, 1, 1, 1

$\because$ LCM of 6, 3, 2 and 8 is $(2\times 2\times 2\times 3)=24$.

$\therefore \frac{-5}{6}=\frac{-5\times 4}{6\times 4}\Rightarrow\frac{-20}{24}; \frac{10}{3}=\frac{10\times 8}{3\times 8}\Rightarrow\frac{80}{24};$

$\frac{-7}{-2}=\frac{-7\times(-12)}{-2\times(-12)}\Rightarrow\frac{84}{24}$ and $\frac{13}{8}=\frac{13\times 3}{8\times 3}\Rightarrow\frac{39}{24};$

Since, $-20<39<80<84$

$\therefore \quad \frac{-20}{24}<\frac{39}{24}<\frac{80}{24}<\frac{84}{24}$

Hence, the ascending order is $\left(\frac{-5}{6}<\frac{13}{8}<\frac{10}{3}<\frac{-7}{-2}\right)$.

4. Arrange in descending order :

(a) $\frac{-7}{10}, \frac{23}{-5}, \frac{-2}{15}, \frac{-11}{30}$

2	10, 5, 15, 30
3	5, 5, 15, 15
5	5, 5, 5, 5
	1, 1, 1, 1

$\because$ LCM of 10, 5, 15 and 30 is $(2\times 3\times 5)=30$.

$\therefore \frac{-7}{10}=\frac{-7\times 3}{10\times 3}\Rightarrow\frac{-21}{30}; \frac{23}{-5}=\frac{23\times(-6)}{-5\times(-6)}\Rightarrow\frac{-138}{30};$

$\frac{-2}{15}=\frac{-2\times 2}{15\times 2}\Rightarrow\frac{-4}{30};$ and $\frac{-11}{30}=\frac{-11}{30};$

Since, $-4>-11>-21>-138$

$\therefore \quad \frac{-4}{30}>\frac{-11}{30}>\frac{-21}{30}>\frac{-138}{30}$

Hence, the descending order is $\left(\frac{-2}{15}>\frac{-11}{30}>\frac{-7}{10}>\frac{23}{-5}\right)$.

(b) $\frac{-11}{5}, \frac{13}{-8}, \frac{-7}{4}, \frac{17}{-10}$

2	5, 8, 4, 10
2	5, 4, 2, 5
2	5, 2, 1, 5
5	5, 1, 1, 5
	1, 1, 1, 1

$\because$ LCM of 5, 8, 4 and 10 is $(2\times 2\times 2\times 5)=40$.

$\therefore \frac{-11}{5}=\frac{-11\times 8}{5\times 8}\Rightarrow\frac{-8}{40}; \frac{13}{-8}=\frac{13\times(-5)}{-8\times(-5)}\Rightarrow\frac{-65}{40};$

$\frac{-7}{4}=\frac{-7\times 10}{4\times 10}\Rightarrow\frac{-70}{40};$ and $\frac{17}{-10}=\frac{17\times(-4)}{-10\times(-4)}$

$\Rightarrow\frac{-68}{40};$

Since, $-65>-68>-70>-88$

$\therefore \frac{-65}{40}>\frac{-68}{40}>\frac{-70}{40}>\frac{-88}{40}$

(c) $\frac{-5}{6}, \frac{13}{-18}, \frac{-7}{12}, \frac{-15}{24}$

$\because$ LCM of 6, 18, 12 and 24 is $(2\times 2\times 2\times 3\times 3)=72$.

$$\therefore \frac{-5}{6} = \frac{-5\times 12}{6\times 12} \Rightarrow \frac{-60}{72};\ \frac{13}{-18} = \frac{13\times(-4)}{-18\times(-4)} = \frac{-52}{72};$$

$$\frac{-7}{12} = \frac{-7\times 6}{12\times 6} \Rightarrow \frac{-42}{72};\ \text{and}\ \frac{-15}{24} = \frac{-15\times 3}{24\times 3} \Rightarrow \frac{-45}{72};$$

2	6, 18, 12, 24
2	3, 9, 6, 12
2	3, 9, 3, 6
3	3, 9, 3, 3
3	1, 3, 1, 1
	1, 1, 1, 1

Since, $-42 > -45 > -52 > -60$

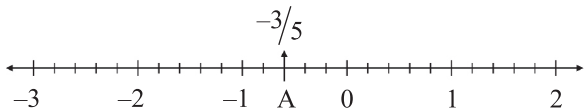
$$\therefore \frac{-42}{72} > \frac{-45}{72} > \frac{-52}{72} > \frac{-60}{72}$$

Hence, the descending order is

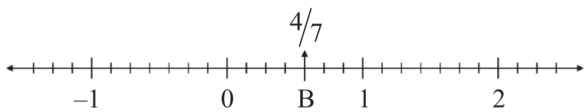
$$\left(\frac{-7}{12} > \frac{-15}{24} > \frac{13}{-18} > \frac{-5}{6}\right).$$

5. Represent the following rational numbers on a number line :

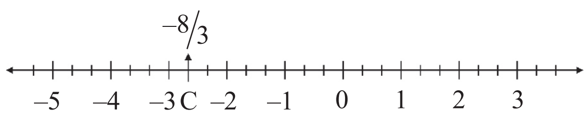
(a) $\frac{-3}{5}$



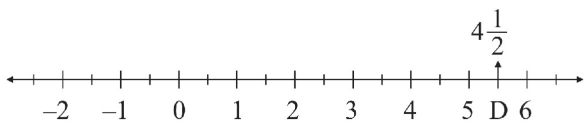
(b) $\frac{4}{7}$



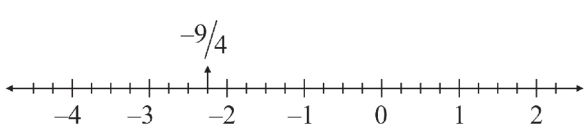
(c) $\frac{-8}{3}$



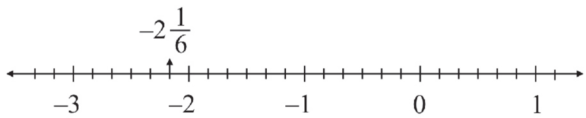
(d) $4\frac{1}{2}$



(e) $-\frac{9}{4}$



(f) $-2\frac{1}{6}$



6. Evaluate :

(a) $\left|\frac{2}{3}\right| - \left|\frac{-7}{5}\right| = \frac{2}{3} - \frac{7}{5} = \frac{10-21}{15} \Rightarrow \frac{-11}{15}.$

(b) $\left|\frac{8}{15}\right| + \left|\frac{-3}{16}\right| = \frac{8}{15} + \frac{3}{16} = \frac{128+45}{240} \Rightarrow \frac{173}{240}.$

(c) $-|-(-2)| + \left|-\frac{1}{6}\right| = -|2| + \frac{1}{6} = -2 + \frac{1}{6} = \frac{-12+1}{6} = \frac{-11}{6}.$

7. Replace by >, < or = to make the statement true :

(a) $\left|\frac{7}{5}\right| \square \left|\frac{-8}{5}\right|$

$\because \left|\frac{7}{5}\right| = \frac{7}{5}$

And, $\left|\frac{-8}{5}\right| = \frac{8}{5}$

Since, $\frac{7}{5} < \frac{8}{5}$

So, $\left|\frac{7}{5}\right| < \left|\frac{-8}{5}\right|$

(b) $\left|\frac{-11}{4}\right| \square \left|\frac{-3}{2}\right|$

$\because \left|\frac{-11}{4}\right| = \frac{-11}{4}$

And, $\left|\frac{-3}{2}\right| = \frac{3\times 2}{2\times 2} \Rightarrow \frac{6}{4}$

Since, $\frac{11}{4} > \frac{6}{4}$

So, $\left|\frac{-11}{4}\right| > \left|\frac{-3}{2}\right|$

(c) $\left|\frac{-8}{12}\right| \square \left|\frac{-10}{-15}\right|$

$\because \left|\frac{-8}{12}\right| = \frac{8}{12} \Rightarrow \frac{2}{3}$

And, $\left|\frac{-10}{-15}\right| = \frac{10}{15} \Rightarrow \frac{2}{3}$

Since, $\frac{2}{3} = \frac{2}{3}$

So, $\left|\frac{-8}{12}\right| = \left|\frac{-10}{15}\right|$

(d) $\left|\frac{-5}{6}\right| \square \left|\frac{6}{-8}\right|$

$\because \left|\frac{-5}{6}\right| = \frac{5\times 4}{6\times 4} \Rightarrow \frac{20}{24}$

And, $\left|\frac{6}{-8}\right| = \frac{6\times 3}{8\times 3} \Rightarrow \frac{18}{24}$

Since, $\frac{20}{24} > \frac{18}{24}$

So, $\left|\frac{-5}{6}\right| > \left|\frac{6}{-8}\right|$

8. Verify that $|x| = |-x|$ for :

(a) $x = \frac{2}{3}$

$\therefore \quad -x = \frac{-2}{3}$

Now, $|x| = \left|\frac{2}{3}\right| \Rightarrow \frac{2}{3}$

And, $|-x| = \left|\frac{-2}{3}\right| \Rightarrow \frac{2}{3}$

Hence, $|x| = |-x|$

(b) $x = \frac{-3}{4}$

$\therefore \quad -x = -\left(\frac{-3}{4}\right) \Rightarrow \frac{3}{4}$

Now, $|x| = \left|\frac{-3}{4}\right| \Rightarrow \frac{3}{4}$

And, $|-x| = \left|\frac{3}{4}\right| \Rightarrow \frac{3}{4}$

Hence, $|x| = |-x|$

(c) $x = \frac{-4}{-3}$

$\therefore \quad -x = \frac{-4}{3}$

Now, $|x| = \left|\frac{4}{3}\right| \Rightarrow \frac{4}{3}$

And, $|-x| = \left|\frac{-4}{3}\right| \Rightarrow \frac{4}{3}$

Hence, $|x| = |-x|$

9. Verify that $|x + y| \le |x| + |y|$ for :

(a) $x = \frac{2}{5}, \ y = \frac{-1}{3}$

Now, $\quad |x + y| = \left|\frac{2}{5} + \frac{-1}{3}\right|$

$= \left|\frac{2}{5} - \frac{1}{3}\right| = \left|\frac{6-5}{15}\right| = \left|\frac{1}{15}\right| \Rightarrow \frac{1}{15}$

And, $|x| + |y| = \left|\frac{2}{5}\right| + \left|\frac{-1}{3}\right|$

$= \frac{2}{5} + \frac{1}{3} = \frac{6+5}{15} \Rightarrow \frac{11}{15}$

$\because \quad \frac{1}{15} < \frac{11}{15}$

$\therefore \quad |x + y| < |x| + |y|$

Hence, $|x + y| \le |x| + |y|$

(b) $x = \frac{7}{-5}, \ y = \frac{-5}{7}$

Now, $\quad |x + y| = \left|\frac{7}{-5} + \frac{-5}{7}\right|$

$= \left|\frac{-7}{5} - \frac{5}{7}\right| = \left|\frac{-49-25}{35}\right| = \left|\frac{-74}{35}\right| \Rightarrow \frac{74}{35}$

And, $|x|+|y|=\left|\frac{7}{-5}\right|+\left|\frac{-5}{7}\right|$

$=\frac{7}{5}+\frac{5}{7}=\frac{49+25}{35}\Rightarrow\frac{74}{35}$

$\because \frac{74}{35}=\frac{74}{35}$

$\therefore |x+y|=|x|+|y|$ **Proved**

10. (a) Given, $x=7$ and $y=3$.

$\therefore |x-y|=|7-3|=|4|\Rightarrow 4$

And, $|y-x|=|3-7|=|-4|\Rightarrow 4$

So, $|x-y|=|y-x|$

Yes, they are equal.

(b) Given, $x=-8$ and $y=2$.

$\therefore |x+y|=|-8+2|=|-6|\Rightarrow 6.$

Exercise 1.2

1. (a) $\frac{6}{-13}$ and $\frac{5}{7}$

$=\frac{-6}{13}+\frac{5}{7}=\frac{-6\times 7+5\times 13}{13\times 7}=\frac{-42+65}{91}=\frac{23}{91}$

(b) $\frac{-7}{12}$ and $\frac{-11}{-18}$

$=\frac{-7}{12}+\frac{-11}{-18}=\frac{-7\times 3+11\times 2}{36}=\frac{-21+22}{36}=\frac{1}{36}$

(c) 5 and $\frac{6}{15}$

$=5+\frac{6}{15}=\frac{5\times 13+6\times 1}{15}=\frac{75+6}{15}=\frac{81}{15}\Rightarrow\frac{27}{5}$

2. (a) $\frac{6}{15}$ from $\frac{-4}{5}$

$=\frac{-4}{5}-\frac{6}{15}=\frac{-4\times 3-6\times 1}{15}=\frac{-12-6}{15}=\frac{-18}{15}\Rightarrow\frac{-6}{5}$

(b) $\frac{-5}{13}$ from $\frac{8}{-7}$

$=\frac{8}{-7}-\left(\frac{-5}{13}\right)=\frac{-8}{7}+\frac{5}{13}=\frac{-8\times 13+5\times 7}{91}=\frac{-104+35}{91}=\frac{-69}{91}$

(c) $\frac{-13}{14}$ from $\frac{11}{20}$

$=\frac{11}{20}-\left(\frac{-13}{14}\right)=\frac{11}{20}+\frac{13}{14}$

$=\frac{11\times 7+13\times 10}{140}=\frac{91+130}{140}$

$= \frac{221}{140}.$

3. (a) $\frac{-7}{9} - \left(\frac{-5}{12}\right) + \frac{1}{3}$

$= \frac{-7}{9} + \frac{5}{12} + \frac{1}{3} = \frac{-7 \times 4 + 5 \times 3 + 1 \times 12}{36}$

$= \frac{-28 + 15 + 12}{36} = \frac{-28 + 27}{36} \Rightarrow \frac{-1}{36}.$

(b) $\frac{11}{18} + \left(\frac{-2}{9}\right) - \frac{3}{16}$

$= \frac{11}{18} - \frac{2}{9} - \frac{3}{16} = \frac{11 \times 8 - 2 \times 16 - 3 \times 9}{144} = \frac{88 - 32 - 27}{144}$

$= \frac{88 - (32 + 27)}{144} = \frac{88 - 59}{144} = \frac{29}{144}.$

(c) $\frac{5}{7} - \frac{11}{6} + \frac{8}{9}$

$= \frac{5 \times 18 - 11 \times 21 + 8 \times 14}{126} = \frac{90 - 231 + 112}{126} = \frac{202 - 231}{126} \Rightarrow \frac{-29}{126}.$

4. Find additive inverse of :

(a) The additive inverse of $\frac{-6}{13}$ is $\frac{6}{13}$.

(b) The additive inverse of $\frac{4}{15}$ is $\frac{-4}{15}$.

(c) The additive inverse of -8 is 8.

(d) The additive inverse of $\frac{-16}{-31}$ is $-\frac{16}{31}$.

5. (a) Given : $x = \frac{-1}{2}, y = \frac{5}{12}, z = \frac{-6}{15}$

$\therefore \quad (x + y) + z = \left(-\frac{1}{2} + \frac{5}{12}\right) + \frac{-6}{15}$

$= \left(\frac{-1 \times 6 + 5}{12}\right) - \frac{6}{15}$

$= \left(\frac{-6 + 5}{12}\right) - \frac{6}{15}$

$= \frac{-1}{12} - \frac{6}{15}$

$= \frac{-1 \times 5 - 6 \times 4}{60}$

$= \frac{-5 - 24}{60} \Rightarrow \frac{-29}{60}.$

And, $x+(6+z)=\frac{-1}{2}+\left(\frac{5}{12}+\frac{-6}{15}\right)$

$$=\frac{-1}{2}+\left(\frac{5}{12}-\frac{6}{15}\right)$$

$$=\frac{-1}{2}+\left(\frac{5\times 5-6\times 4}{60}\right)$$

$$=\frac{-1}{2}+\left(\frac{25-24}{60}\right)$$

$$=\frac{-1}{2}+\frac{1}{60}$$

$$=\frac{-1\times 30+1}{60}$$

$$=\frac{-30+1}{60}\Rightarrow\frac{-29}{60}$$

Hence, $[(x+y)+z=x+(y+z)]$ **Verified.**

(b) Given : $x=3,\ y=\frac{-2}{5},\ z=\frac{-7}{10}$

$\therefore \quad (x+y)+z=\left(3+\frac{-2}{5}\right)+\frac{-7}{10}$

$$=\left(3-\frac{2}{5}\right)-\frac{7}{10}$$

$$=\left(\frac{3\times 5-2\times 1}{5}\right)-\frac{7}{10}$$

$$=\left(\frac{15-2}{5}\right)-\frac{7}{10}$$

$$=\frac{13}{5}-\frac{7}{10}$$

$$=\frac{13\times 2-7\times 1}{10}$$

$$=\frac{26-7}{10}\Rightarrow\frac{19}{20}$$

And, $x+(y+z)=3+\left(\frac{-2}{5}+\frac{-7}{10}\right)$

$$=3+\left(\frac{-2}{5}-\frac{7}{10}\right)=3+\left(\frac{-2\times 2-7\times 1}{10}\right)$$

$$=3+\left(\frac{-4-7}{10}\right)=3+\left(\frac{-11}{10}\right)$$

$$=3+\left(\frac{-11}{10}\right)$$

$$= 3 - \frac{11}{10}$$

$$= \frac{3 \times 10 - 11 \times 1}{10}$$

$$= \frac{30 - 11}{10} \Rightarrow \frac{19}{10}$$

Hence, $[(x + y) + z = x + (y + z)]$ **Verified.**

6. (a) $\frac{-6}{5} + \frac{3}{14} + \frac{-6}{7} + \frac{7}{15}$

$$= \frac{3}{14} + \frac{-6}{7} + \frac{7}{15} + \frac{-6}{5}$$

$$= \frac{3}{14} - \frac{6}{7} + \frac{7}{15} - \frac{6}{5}$$

$$= \frac{3-12}{14} + \frac{7-18}{15}$$

$$= \frac{-9}{14} + \frac{-11}{15}$$

$$= \frac{-9}{14} - \frac{11}{15}$$

$$= \frac{-135-154}{210}$$

$$= \frac{-289}{210}.$$

(b) $\frac{-1}{2} + \frac{7}{4} + \frac{-5}{6} + \frac{11}{6}$

$$= \frac{-1}{2} + \frac{7}{4} - \frac{5}{6} + \frac{11}{6}$$

$$= \frac{-1 \times 2 + 7 \times 1}{4} + \frac{-5 + 11}{6}$$

$$= \frac{-2+7}{4} + \frac{6}{6}$$

$$= \frac{5}{4} + 1$$

$$= \frac{5+4}{4} \Rightarrow \frac{9}{4}.$$

(c) $\frac{7}{9} + \frac{-2}{3} + \frac{-11}{18} + \frac{1}{6}$

$$= \frac{7}{9} - \frac{2}{3} - \frac{11}{18} + \frac{1}{6}$$

$$= \frac{7 \times 2 - 2 \times 6 - 11 \times 1 + 1 \times 3}{18}$$

$$= \frac{14 - 12 - 11 + 3}{18}$$

$$= \frac{14 + 3 - (12 + 11)}{18}$$

$$= \frac{17 - 23}{18}$$

$$= \frac{-6}{18} \Rightarrow \frac{-1}{3}$$

7. The sum of two numbers $= \frac{21}{8}$.

And one of them $= \frac{-2}{5}$

Thus, the other number = sum of two numbers – one number

$$= \frac{21}{8} - \left(\frac{-2}{5}\right)$$

$$= \frac{21}{8} + \frac{2}{5}$$

$$= \frac{21 \times 5 + 2 \times 8}{40}$$

$$= \frac{105 + 16}{40} \Rightarrow \frac{121}{40}$$

Hence, the another rational number is $\frac{121}{40}$.

8. The sum of two numbers $= \frac{-15}{16}$

And, one of the number $= \frac{-7}{12}$

Thus, the other number = sum of two numbers – one number.

$$= \frac{-15}{16} - \left(\frac{-7}{12}\right)$$

$$= \frac{-15}{16} + \frac{7}{12}$$

$$= \frac{-15 \times 3 + 7 \times 4}{48}$$

$$= \frac{-45 + 28}{48} \Rightarrow \frac{-17}{48}$$

Hence, the another rational number is $\frac{-17}{48}$.

9. Let, X should be added.

$$\therefore \quad \frac{-3}{8} + X = \frac{7}{9}$$

$$X = \frac{7}{9} + \frac{3}{8}$$

$$= \frac{56 + 27}{72} = \frac{83}{72}$$

Hence, $\frac{83}{72}$ added to $\frac{-3}{8}$ to make the sum $\frac{7}{9}$.

10. Let, Y should be subtracted.

$$\therefore \quad \frac{-9}{2} - Y = 1$$

$$\frac{-9}{2} - 1 = Y$$

$$\frac{-9-2}{2} = Y$$

$$\frac{-11}{2} = Y$$

Hence, $\frac{-11}{2}$ subtracted from $\frac{-9}{2}$ to get 1.

11. $\left[\text{Sum of } \frac{3}{-5} \text{ and } \frac{8}{15}\right] - \left[\text{Sum of } \frac{-5}{8} \text{ and } \frac{7}{10}\right]$

$$= \left[\frac{3}{-5} + \frac{8}{15}\right] - \left[\frac{-5}{8} + \frac{7}{10}\right]$$

$$= \left[\frac{-3\times 3 + 8\times 1}{15}\right] - \left[\frac{-5\times 5 + 7\times 4}{40}\right]$$

$$= \left[\frac{-9+8}{15}\right] - \left[\frac{-25+28}{40}\right]$$

$$= \left[\frac{-1}{15}\right] - \left[\frac{3}{40}\right]$$

$$= \frac{-1}{15} - \frac{3}{40}$$

$$= \frac{-1\times 8 - 3\times 3}{120}$$

$$= \frac{-8-9}{120} \Rightarrow \frac{-17}{120}.$$

12. Let, A should be added.

$$\therefore \quad \left(\frac{-5}{7} + \frac{3}{8}\right) + A = \frac{-9}{4}$$

$$A = \frac{-9}{4} - \left(\frac{-5}{7} + \frac{3}{8}\right)$$

$$= -\frac{9}{4} + \frac{5}{7} - \frac{3}{8}$$

$$= \frac{-9\times 14 + 5\times 8 - 3\times 7}{56}$$

$$= \frac{-126 + 40 - 21}{56}$$

$$= \frac{-126 + 40 - 21}{56}$$

$$= \frac{-147 + 40}{56} \Rightarrow \frac{-107}{56}$$

Hence, $\frac{-107}{56}$ added to $\left(\frac{-5}{7} + \frac{3}{8}\right)$ to obtain $\frac{-9}{4}$.

13. Total length of a piece of wire $= \frac{15}{4}$ m

And, length of one broken piece $= 2\frac{1}{2}$ m

$\therefore$ Length of other piece of wire $= \frac{15}{4}\text{ m} - 2\frac{1}{2}\text{ m.}$

$= \frac{15}{4}\text{ m} - \frac{5}{2}\text{ m}$

$= \left(\frac{15-10}{4}\right)\text{ m} = \frac{5}{4}\text{ m or } 1\frac{1}{4}\text{ m}$

Hence, the length of other broken piece of wire is $1\frac{1}{4}$ m.

14. Time duration of a TV show $= 2\frac{1}{2}5$ hours

And, time spent on advertisements $= 1\frac{1}{4}$ hours.

Thus, actual time duration of TV show $= 2\frac{1}{2}\text{ hours} - 1\frac{1}{4}\text{ hours}$

$= \left(\frac{5}{2} - \frac{5}{4}\right)\text{ hours}$

$= \left(\frac{10-5}{4}\right)\text{ hours}$

$= \frac{5}{4}\text{ hr} \Rightarrow 1\frac{1}{4}\text{ hours}$

Hence, the actual time duration of the TV show is $1\frac{1}{4}$ hours.

Exercise 1.3

1. (a) The multiplicative inverse of $\frac{11}{29}$ is $\frac{29}{11}$.

(b) The multiplicative inverse of $\frac{-16}{23}$ is $\frac{-23}{16}$.

(c) The multiplicative inverse of -13 is $\frac{-1}{13}$.

(d) The multiplicative inverse of $2\frac{1}{3}$ or $\frac{7}{3}$ is $\frac{3}{7}$.

2. (a) $-\frac{4}{5} \times \frac{5}{7} \times \left(-\frac{8}{9}\right) + \frac{8}{9} \times \frac{4}{7}$

$= -\frac{4}{7} \times \left(-\frac{8}{9}\right) + \frac{32}{63} = \frac{+32}{63} + \frac{32}{63} = \frac{32+32}{63} \Rightarrow \frac{64}{63}$

(b) $\frac{2}{13}\times\left(-\frac{5}{7}+\frac{1}{7}+\frac{4}{7}\right)$

$=\frac{2}{13}\times\left[\frac{-5+1+4}{7}\right]=\frac{2}{13}\times\left[\frac{-5+5}{7}\right]=\frac{2}{13}\times\frac{0}{7}\Rightarrow 0$

(c) $\left(-\frac{3}{13}\right)\times\left(-\frac{2}{13}\right)\times\left(-\frac{1}{13}\right)\times 0\times\frac{1}{13}\times\frac{2}{13}\times\frac{3}{13}=0.$

3. (a) $\frac{3}{7}\times 2\frac{1}{3}\times\frac{3}{7}\times 1\frac{2}{3}$

$=\frac{3}{7}\times\left[2\frac{1}{3}+1\frac{2}{3}\right]$

$=\frac{3}{7}\times\left[\frac{7}{3}+\frac{5}{3}\right]$

$=\frac{3}{7}\times\left[\frac{7+5}{3}\right]$

$=\frac{3}{7}\times\frac{12}{3}\Rightarrow\frac{12}{7}$

(b) $\frac{4}{7}\times\frac{10}{9}-\frac{4}{7}\times\frac{1}{9}$

$=\frac{4}{7}\times\left[\frac{10}{9}-\frac{1}{9}\right]$

$=\frac{4}{7}\times\left[\frac{10-1}{9}\right]$

$=\frac{4}{7}\times\frac{9}{9}\Rightarrow\frac{4}{7}$

(c) $-\frac{4}{7}\times\frac{5}{8}+\left(\frac{4}{5}\right)\times\left(-\frac{7}{8}\right)$

$=-\frac{4}{8}\times\left[\frac{5}{7}+\frac{7}{5}\right]$

$=\frac{-4}{2}\times\left[\frac{25+49}{35}\right]$

$=\frac{-1}{2}\times\frac{74}{35}$

$=\frac{-37}{35}$

(d) $\frac{5}{11}\times\left(\frac{-6}{11}\right)+(-4)\times\frac{5}{11}$

$=\frac{5}{11}\times\left[\left(\frac{-6}{11}\right)+(-4)\right]$

$=\frac{5}{11}\times\left[\frac{-6}{11}-4\right]$

$=\frac{5}{11}\times\left[\frac{-6-44}{11}\right]$

$=\frac{5}{11}\times\left(\frac{-50}{11}\right)$

$=\frac{-250}{121}$

4. (a) $\frac{-7}{9}\times\left(\frac{-14}{25}\times\frac{15}{16}\right)=\left(\frac{-7}{9}\times\frac{-14}{25}\right)\times\frac{15}{16}$

LHS $=\frac{-7}{9}\times\left(\frac{-14}{25}\times\frac{15}{16}\right)$

$=\frac{-7}{9}\times\left(\frac{-7\times 3}{5\times 8}\right)$

$=\frac{-7}{9}\times\frac{-21}{40}$

$=\frac{-7\times(-7)}{3\times 40}\Rightarrow\frac{49}{120}$

And, RHS $= \left(\frac{-7}{9} \times \frac{-14}{25}\right) \times \frac{15}{16}$

$= \frac{98}{225} \times \frac{15}{16}$

$= \frac{49 \times 1}{15 \times 8} \Rightarrow \frac{49}{120}$

$\therefore$ LHS = RHS

Hence, Associative Property of Multiplication is verified.

(b) $\left(\frac{-1}{21} \times \frac{14}{15}\right) \times \frac{25}{-26} = \frac{-1}{21} \times \left(\frac{14}{15} \times \frac{25}{-26}\right)$

LHS $= \left(\frac{-1}{21} \times \frac{14}{15}\right) \times \frac{25}{-26}$

$= \left(\frac{-1 \times 2}{3 \times 15}\right) \times \frac{25}{-26}$

$= \left(\frac{-2}{45}\right) \times \left(\frac{25}{-20}\right)$

$= \frac{5}{9 \times 13} \Rightarrow \frac{5}{117}$

And, RHS $= -\frac{1}{21} \times \left(\frac{14}{15} \times \frac{25}{-26}\right)$

$= \frac{-1}{21} \times \left(\frac{7 \times 5}{3 \times (-13)}\right)$

$= \left(\frac{-1}{21}\right) \times \left(\frac{35}{-39}\right)$

$= \frac{(-1) \times 5}{3 \times (-39)} \Rightarrow \frac{5}{117}$

$\therefore$ LHS = RHS.

Hence, Associative Property of Multiplication is verified.

(c) $-56 \times \frac{2}{7} = \frac{2}{7} \times -56$

LHS $= -56 \times \frac{2}{7}$

$= (-8) \times 2 \Rightarrow -16$

And, RHS $= \frac{2}{7} \times -56$

$= 2 \times (-8) \Rightarrow -16$

$\therefore$ LHS = RHS

Hence, Commutative property of multiplication is verified.

5. (a) $\frac{-3}{8} \times \left(\frac{-6}{11} + \frac{4}{9}\right) = \left(\frac{-3}{8} \times \frac{-6}{11}\right) + \left(\frac{-3}{8} \times \frac{4}{9}\right)$

LHS $= \frac{-3}{8} \times \left(\frac{-6}{11} + \frac{4}{9}\right)$

$= \frac{-3}{8} \times \left(\frac{-54+44}{99}\right)$

$= \frac{-3}{8} \times \left(\frac{-10}{99}\right)$

$= \frac{(-1) \times (-5)}{4 \times 33} \Rightarrow \frac{5}{132}.$

And, RHS $= \left(\frac{-3}{8} \times \frac{-6}{11}\right) + \left(\frac{-3}{8} \times \frac{4}{9}\right)$

$= \left(\frac{18}{88}\right) + \left(\frac{-12}{72}\right) = \frac{9}{44} - \frac{1}{6}$

$= \frac{27-22}{132} \Rightarrow \frac{5}{132}$

$\therefore$ LHS = RHS

Hence, Distributive property of multiplication over addition is verified.

(b) $\frac{8}{7} \times \left(\frac{-10}{9} + \frac{1}{3}\right) = \left(\frac{8}{7} \times \frac{-10}{9}\right) + \left(\frac{8}{7} \times \frac{-1}{3}\right)$

LHS $= \frac{8}{7} \times \left(\frac{-10}{9} + \frac{1}{3}\right)$

$= \frac{8}{7} \times \left(\frac{-10+3}{9}\right)$

$= \frac{8}{7} \times \left(\frac{-7}{9}\right)$

$= \frac{8 \times (-1)}{1 \times 9} \Rightarrow \frac{-8}{9}$

And, RHS $= \left(\frac{8}{7} \times \frac{-10}{9}\right) + \left(\frac{8}{7} \times \frac{-1}{3}\right)$

$= \frac{-80}{63} + \frac{8}{21}$

$= \frac{-80 + 8 \times 3}{63} = \frac{-80+24}{63}$

$= \frac{-56}{63} \Rightarrow \frac{-8}{9}$

$\therefore$ LHS = RHS

Hence, distributive property of multiplication over Addition is verified.

6. The cost of one litre of petrol $= ₹\ 42\frac{3}{7}$.

$\therefore$ the cost of 28 litres of petrol $= ₹\ 42\frac{3}{7} \times 28 = ₹\ \frac{297}{7} \times 28$

$= ₹\ 297 \times 4 \Rightarrow ₹\ 1188$

Hence, the cost of 28 litres of petrol is ₹ 1188.

7. Length of a rectangular park $= 15\frac{2}{5}$ m

And, Breadth of the rectangular park $= 9\frac{1}{6}$ m.

So, the area of the rectangular park = length × breadth

$$= 15\frac{2}{5}\text{ m} \times 9\frac{1}{6}\text{ m.}$$

$$= \frac{77}{5}\text{ m} \times \frac{55}{6}\text{ m}$$

$$= \frac{77 \times 11}{6}\text{ m}^2$$

$$= \frac{847}{6}\text{ m}^2$$

$$= 141\frac{1}{6}\text{ m}^2$$

Hence, the area of the rectangular park is $141\frac{1}{6}$ sq. m.

8. Each side of a square piece of land $= 6\frac{1}{4}$ m.

$\therefore$ Area of the square piece of land $= (\text{side})^2$

$$= \left(6\frac{1}{4}\text{ m}\right)^2 = \left(\frac{25}{4}\text{ m}\right)^2$$

$$= \frac{625}{16}\text{ m}^2 = 39\frac{1}{16}\text{ m}^2.$$

Hence, the area of the square piece of land is $39\frac{1}{16}$ sq. m.

Exercise 1.4

1. (a) $\frac{-36}{55} \div \frac{-3}{10}$

$\frac{-36}{55} \times \frac{10}{-3} = \frac{12 \times 2}{11} \Rightarrow \frac{24}{11}$

(b) $\frac{-2}{21} \div \frac{7}{-14}$

$= \frac{-2}{21} \times \frac{-14}{7}$

$= \frac{-2 \times (-2)}{21} \Rightarrow \frac{4}{21}$

(c) $-7 \div \frac{5}{13}$

$= -7 \times \frac{13}{5} = \frac{-91}{5}$

(d) $-2\frac{3}{8} \div \frac{35}{42}$

$= \frac{-19}{8} \times \frac{42}{35}$

$= \frac{-19}{8} \times \frac{6}{5} = \frac{-19 \times 3}{4 \times 5} \Rightarrow \frac{-57}{20}$

2. The product of two numbers $\frac{-56}{85}$.

And, one of the number $\frac{35}{-63}$.

Thus, the other number = Product ÷ one number.

$= \frac{-56}{85} \div \frac{35}{-63}$

$= \frac{-56}{85} \times \frac{-63}{35}$

$= \frac{-8 \times (-63)}{85 \times 5} = \frac{504}{425}$

Hence, the other rational number is $\frac{504}{425}$.

3. The product of two rational numbers = $\frac{-72}{121}$.

And, one of the number = $\frac{88}{27}$

Thus, the other number = Product ÷ one number

$= \frac{-72}{121} \div \frac{88}{27}$

$= \frac{-72}{121} \times \frac{27}{88}$

$= \frac{-9 \times 27}{121 \times 11} = \frac{-243}{1331}$

Hence, the other rational number is $\frac{-243}{1331}$.

4. Let, $\frac{-12}{13}$ multiplied by x.

$\therefore \quad \frac{-12}{13} \times x = \frac{4}{39}$

$x = \frac{4}{39} \times \frac{13}{-12}$

$= \frac{1}{3} \times \frac{1}{-3} \Rightarrow \frac{-1}{9}$

Hence, $\frac{-12}{13}$ multiplied by $\frac{-1}{9}$ to get $\frac{4}{39}$.

5. Let, $\frac{-54}{15}$ divided by x.

Thus, $\frac{-54}{15} \div x = \frac{-42}{35}$

$$\frac{-54}{15} \times \frac{1}{x} = \frac{-42}{35}$$

$$\frac{-54}{15} \times \frac{35}{-42} = x$$

$$\frac{9 \times 7}{3 \times 7} = x$$

$$3 = x$$

$$x = 3$$

6. $\left[\text{Sum of } \frac{11}{7} \text{ and } \frac{-7}{5}\right] \div \left[\text{Product of } \frac{11}{7} \text{ and } \frac{-7}{5}\right]$

$$= \left[\frac{11}{7} + \frac{-7}{5}\right] \div \left[\frac{11}{7} \times \frac{-7}{5}\right]$$

$$= \left[\frac{11}{7} - \frac{7}{5}\right] \div \left[\frac{-77}{35}\right]$$

$$= \left[\frac{55-49}{35}\right] \div \left(\frac{-11}{5}\right)$$

$$= \frac{6}{35} \times \frac{5}{-11}$$

$$= \frac{6}{7 \times (-11)} \Rightarrow \frac{-6}{77}$$

7. $\left[\text{Sum of } \frac{-9}{4} \text{ and } \frac{-8}{3}\right] \div \left[\text{Difference of } \frac{13}{8} \text{ and } \frac{-7}{16}\right]$

$$= \left[\frac{-9}{4} + \frac{-8}{3}\right] \div \left[\frac{13}{8} - \left(\frac{-7}{16}\right)\right]$$

$$= \left[\frac{-9}{4} - \frac{8}{3}\right] \div \left[\frac{13}{8} + \frac{7}{16}\right]$$

$$= \left[\frac{-27-32}{12}\right] \div \left[\frac{26+7}{16}\right]$$

$$= \left[\frac{-59}{12}\right] \div \frac{33}{16}$$

$$= \frac{-59}{12} \times \frac{16}{33}$$

$$= \frac{-59 \times 4}{3 \times 33} \Rightarrow \frac{-236}{99}$$

8. (a) $\frac{7}{12} \div x = \frac{-14}{3}$

$\frac{7}{12} \times \frac{1}{x} = \frac{-14}{3}$

$\frac{7 \times 3}{12 \times (-14)} = x$

$\frac{1 \times 1}{4 \times (-2)} = x$

$x = \frac{1}{-8}$

So, $\frac{7}{12} \div \frac{\mathbf{-1}}{\mathbf{8}} = \frac{-14}{3}$

(b) $x \div \left(-\frac{6}{5}\right) = \frac{15}{26}$

$x \times \frac{5}{-6} = \frac{15}{26}$

$x = \frac{15 \times (-6)}{26 \times 5}$

$x = \frac{3 \times (-3)}{13 \times 1} \Rightarrow \frac{-9}{13}$

So, $\frac{-9}{13} \div \left(-\frac{6}{5}\right) = \frac{15}{26}$

(c) $-25 \div x = -\frac{5}{6}$

$-25 \times \frac{1}{x} = -\frac{5}{6}$

$\frac{-25 \times 6}{-5} = x$

$5 \times 6 = x$

$x = 30$

So, $-25 \div \frac{30}{1} = -\frac{5}{6}$

(d) $x \div (-13) = \frac{-4}{11}$

$x \times \frac{1}{-13} = \frac{-4}{11}$

$x = \frac{-4 \times (-13)}{11}$

$x = \frac{52}{11}$

So, $\frac{\mathbf{52}}{\mathbf{11}} \div (-13) = \frac{-4}{11}$

9. The area of a rectangle = $45\frac{1}{4}$ m^2.

And, the length of the rectangle = $25\frac{3}{8}$ m

∵ Area of the rectangle $= l \times b$

∴ Breadth of the rectangle = Area ÷ length

$= 45\frac{1}{4} \text{ m}^2 \div 25\frac{3}{8} \text{ m}$

$= \frac{181}{4} \text{ m}^2 \div \frac{203}{8} \text{ m}$

$= \frac{181}{4} \times \frac{8}{203} \text{ m}$

$= \frac{181 \times 2}{203} \text{ m} \Rightarrow \frac{362}{203} \text{ m}$

Hence, the breadth of the rectangle is $\frac{362}{203}$ metres.

10. The cost of = $5\frac{2}{7}$ metres of cloth is = ₹ $28\frac{1}{3}$.

Thus, th cost of one metre of cloth = ₹ $28\frac{1}{3} \div 5\frac{2}{7}$

$$= ₹\ \frac{85}{3} \div \frac{37}{7}$$

$$= ₹\ \frac{85}{3} \times \frac{7}{37} = ₹\ \frac{595}{111}$$

Hence, the cost of one metre of clothe is ` $\frac{595}{111}$.

11. Total length of a wire = $64\frac{1}{6}$ m

And, the number of cutting pieces of wire = 5

Thus, the length of each piece of wire $= \left(64\frac{1}{6} \div 5\right)$ m

$$= \left(\frac{385}{6} \times \frac{1}{5}\right) \text{m} = \frac{77}{6} \text{ m} \Rightarrow 12\frac{5}{6} \text{ m}$$

Hence, the length of each cutting piece of wire is 12 $\frac{5}{6}$ m.

12. Which of the following statements are true?

(a) $\frac{-13}{25} \div \frac{4}{-19} = \frac{-4}{19} \div \frac{-13}{25}$

LHS $= \frac{-13}{25} \div \frac{4}{-19}$

$= \frac{-13}{25} \times \frac{-19}{4}$

$= \frac{247}{100}]$

And, RHS $= \frac{-4}{19} \div \frac{-13}{25}$

$= \frac{-4}{19} \times \frac{25}{-13}$

$= \frac{-100}{-247} \Rightarrow \frac{100}{247}$

Since, LHS ≠ RHS.

So this statement is not true.

(b) $\left(\frac{4}{7} \div \frac{2}{9}\right) \div \frac{11}{13} = \frac{4}{7} \div \left(\frac{2}{9} \div \frac{11}{13}\right)$

LHS $= \left(\frac{4}{7} \div \frac{2}{9}\right) \div \frac{11}{3}$

$= \left(\frac{4}{7} \times \frac{9}{2}\right) \div \frac{11}{13}$

$= \left(\frac{2 \times 9}{7}\right) \times \frac{13}{11}$

$= \frac{18}{7} \times \frac{13}{11} \Rightarrow \frac{234}{77}$

And, RHS $= \frac{4}{7} \div \left(\frac{2}{9} \div \frac{11}{13}\right)$

$= \frac{4}{7} \div \left(\frac{2}{9} \times \frac{13}{11}\right)$

$= \frac{4}{7} \div \left(\frac{26}{99}\right) = \frac{4}{7} \times \frac{99}{26}$

$= \frac{2 \times 99}{7 \times 13} \Rightarrow \frac{198}{91}$

Since, LHS ≠ RHS.

So, this statement is not true.

(c) $\frac{5}{-13} \div \frac{-2}{7} = \frac{-2}{7} \times \frac{5}{-13}$

LHS $= \frac{5}{-13} \div \frac{-2}{7}$

$= \frac{5}{-13} \times \frac{7}{-2} \Rightarrow \frac{35}{26}$

And, RHS $= \frac{-2}{7} \times \frac{5}{-13}$

$= \frac{-10}{-91} \Rightarrow \frac{10}{91}$

Since, LHS ≠ RHS.

So, this statement is not true.

(d) $\left[(-17) \div \frac{8}{5}\right] \div \frac{-9}{19} = (-17) \div \left[\frac{8}{5} \div \frac{-9}{19}\right]$

LHS $= \left[(-17) \div \frac{8}{5}\right] \div \frac{-9}{19}$

$= \left[-17 \times \frac{5}{8}\right] \div \frac{-9}{19}$

$= \frac{-85}{8} \times \frac{19}{-9}$

$= \frac{-1615}{-72} \Rightarrow$

And, RHS $= (-17) \div \left[\frac{8}{5} \div \frac{-9}{19}\right]$

$= (-17) \div \left[\frac{8}{5} \times \frac{19}{-9}\right]$

$= (-17) \div \left[\frac{152}{-45}\right]$

$= \frac{-17 \times (-45)}{152} \Rightarrow \frac{765}{152}$

∵ LHS ≠ RHS.

So, this statement is not true.

Exercise 1.5

1. ∵ $\frac{-2}{7} < \frac{-1}{7} < \frac{0}{7} < \frac{1}{7} < \frac{2}{7}$.

Hence, $\frac{-1}{7}, \frac{0}{7}$ and $\frac{1}{7}$ are three rational numbers between $\frac{-2}{7}$ and $\frac{2}{7}$.

2. $\because \dfrac{-1}{3}=\dfrac{-1\times 3}{3\times 3}\Rightarrow \dfrac{-3}{9}$ and $\dfrac{4}{9}=\dfrac{4}{9}$

Since, $\dfrac{-3}{9}<\dfrac{-2}{9}<\dfrac{-1}{9}<\dfrac{0}{9}<\dfrac{1}{9}<\dfrac{2}{9}<\dfrac{3}{9}<\dfrac{4}{9}$

Hence, $\dfrac{-2}{9},\dfrac{-1}{9},\dfrac{2}{9}$ and $\dfrac{3}{9}$ are four rational numbers between $\dfrac{-1}{3}$ and $\dfrac{4}{9}$.

3. A rational number between 3 and 4 $=\dfrac{3+4}{2}\Rightarrow \dfrac{7}{2}$.

4. $\because \dfrac{2}{9}=\dfrac{2\times 8}{9\times 8}\Rightarrow \dfrac{16}{72}$ and $\dfrac{5}{8}=\dfrac{5\times 9}{8\times 9}\Rightarrow \dfrac{45}{72}$.

Since, $\dfrac{16}{72}<\dfrac{17}{72}<\dfrac{18}{72}\ \ldots\ldots\ldots\ \dfrac{43}{72}<\dfrac{44}{72}<\dfrac{45}{72}$

Hence, $\dfrac{21}{72},\dfrac{22}{72},\dfrac{23}{72},\dfrac{40}{72},\dfrac{41}{72}$ and $\dfrac{42}{72}$ are six rational numbers between $\dfrac{2}{9}$ and $\dfrac{5}{9}$.

5. $\because -\dfrac{1}{4}=\dfrac{-1\times 4}{4\times 4}\Rightarrow \dfrac{-4}{16}$ and $\dfrac{3}{8}=\dfrac{3\times 2}{8\times 2}\Rightarrow \dfrac{6}{16}$.

Since, $\dfrac{-4}{10}<\dfrac{-3}{10}<\dfrac{-2}{10}<\dfrac{-1}{10}<\dfrac{0}{10}\ \ldots\ldots\ldots\ \dfrac{4}{16}<\dfrac{5}{16}<\dfrac{6}{16}$.

Hence, $\dfrac{-2}{16},\dfrac{-1}{16},\dfrac{3}{16},\dfrac{4}{16}$ and $\dfrac{5}{16}$ are five rational numbers between $-\dfrac{1}{4}$ and $\dfrac{3}{8}$.

6. $\because \dfrac{2}{3}=\dfrac{2\times 15}{3\times 15}\Rightarrow \dfrac{30}{45}$ and $\dfrac{2}{5}=\dfrac{2\times 9}{5\times 9}\Rightarrow \dfrac{18}{45}$

Since, $\dfrac{18}{45}<\dfrac{19}{45}<\dfrac{20}{45}\ \ldots\ldots\ldots\ -\dfrac{28}{45}<\dfrac{29}{45}<\dfrac{30}{45}$.

Hence, $\dfrac{19}{45},\dfrac{20}{45},\dfrac{21}{45},\dfrac{22}{45},\dfrac{23}{45},\dfrac{24}{45},\dfrac{26}{45},\dfrac{27}{45}$ and $\dfrac{28}{45}$ are ten rational numbers between $\dfrac{2}{3}$ and $\dfrac{2}{5}$.

7. (a) $x=\dfrac{-3}{8}$

$\therefore\ |x|=\left|\dfrac{-3}{8}\right|=\dfrac{3}{8}$

Now, $x=\dfrac{-3}{8}=\dfrac{-3\times 2}{8\times 2}\Rightarrow \dfrac{-6}{16}$

And, $|x|=\dfrac{3}{8}=\dfrac{3\times 2}{8\times 2}\Rightarrow \dfrac{-6}{16}$

Since, $\dfrac{-6}{16}<\dfrac{-5}{16}<\dfrac{-4}{16}<\dfrac{-3}{16}\ \ldots\ldots\ldots\ \dfrac{3}{16}<\dfrac{4}{16}<\dfrac{5}{10}<\dfrac{6}{16}$.

Hence, $\dfrac{-5}{16},\dfrac{-4}{16},\dfrac{-3}{16},\dfrac{-2}{16},\dfrac{-1}{16},\dfrac{1}{16},\dfrac{2}{16},\dfrac{3}{16},\dfrac{4}{16}$ and $\dfrac{5}{16}$ are ten rational numbers between x and $|x|$.

(b) $x = \frac{-5}{7}$

$\therefore \ |x| = \left|\frac{-5}{7}\right| = \frac{5}{7}$

Now, $x = \frac{-5}{7} = \frac{-5 \times 2}{7 \times 2} \Rightarrow \frac{-10}{14}$

And, $|x| = \frac{5}{7} = \frac{5 \times 2}{7 \times 2} \Rightarrow \frac{10}{14}$

Since, $\frac{-10}{14} < \frac{-9}{14} < \frac{-8}{14}$ .... $\frac{9}{14} < \frac{10}{14}$.

Hence, $\frac{-8}{14}, \frac{-7}{14}, \frac{-6}{14}, \frac{-5}{14}, \frac{-4}{14}, \frac{1}{14}, \frac{2}{14}, \frac{3}{14}, \frac{4}{14}$ and $\frac{5}{14}$ are ten rational numbers between x and $|x|$.

MCQ's

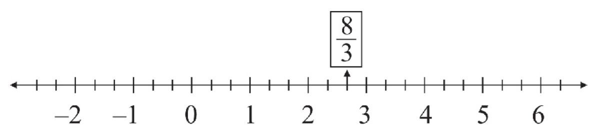
1. (d) **2.** (c) **3.** (b) **4.** (b) **5.** (a) **6.** (b)

Brain Teaser

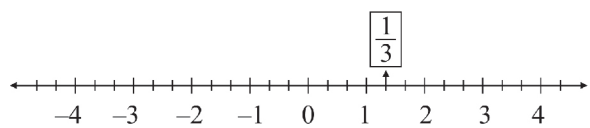
1. Zero has **no** reciprocal.
2. The product of a rational number and its reciprocal is $-\mathbf{1}$.
3. Multiplicative inverse of -1 is $-\mathbf{1}$.
4. **1** is called the multiplicative identity for rationals.
5. The reciprocal of a positive rational number is **Positive** and that of negative rational number is **Negative**.
6. The numbers $-\mathbf{1}$ and **1** are their own reciprocals.
7. The reciprocal of $\frac{1}{x}$, where $x \neq 0$ is $\boldsymbol{x}$.
8. A rational number between $-\frac{1}{5}$ and $\frac{1}{6}$ is **0**.

HOTS

1. (a) $\frac{8}{3}$



(b) $\frac{1}{3}$



2. Given, $x = \frac{4}{5},\ y = \frac{7}{9}$

Thus, $(x+y)^{-1} = \frac{1}{(x+y)}$

$= \frac{1}{\left(\frac{4}{5}+\frac{7}{9}\right)} = \frac{1}{\left(\frac{36+35}{45}\right)} \Rightarrow \frac{45}{71}$

And, $x^{-1} + y^{-1} = \frac{1}{\left(\frac{4}{5}\right)} + \frac{1}{\left(\frac{7}{9}\right)}$

$= \frac{5}{4} + \frac{9}{7}$

$= \frac{35+36}{28} \Rightarrow \frac{71}{28}$

$\because \quad (x+y)^{-1} = \frac{45}{71} < 1$

And, $(x^{-1} + y^{-1}) = \frac{71}{28} > 1$

Hence, a rational number between $(x+y)^{-1}$ and $(x^{-1}+y^{-1})$ is 1.

3. Value of $\left(\frac{9}{13}+\frac{3}{10}\right) = \frac{90+39}{130} \Rightarrow \frac{129}{130}$

And, value of $\left(\frac{3}{10}+\frac{9}{13}\right) = \frac{39+90}{130} \Rightarrow \frac{129}{130}$

$\therefore \quad \frac{9}{13}+\frac{3}{10} = \frac{3}{10}+\frac{9}{13}$

Yes, these two results are same.
Hence, commutative property of addition is verified.

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2 Linear Equations in One Variable

Exercise 2.1

1. (a) $3x - 5 = 20 - 2x$

$3x + 2x = 20 + 5$

$5x = 25$

$x = \frac{25}{5} \Rightarrow 5$

$\therefore \quad x = 5$

Hence, $(x = 5)$ is the solution of the given equation.

Check : $3x - 5 = 3 \times 5 - 5$

$= 15 - 5$

$= 10$

And, $20 - 2x = 20 - 2 \times 5$

$= 20 - 10$

$= 10$

$\therefore$ LHS = RHS.

(b) $21x-2=5x-4$

$21x-5x=-4+2$

$16x=-2$

$x=\frac{-2}{16}\Rightarrow\frac{-1}{8}$

$x=\frac{-1}{8}$

Hence, $\left(x=\frac{-1}{8}\right)$ is the solution of the given equation.

Check : $21x-2=21\times\left(\frac{-1}{8}\right)-2$

$=\frac{-21}{8}-2$

$=\frac{-21-16}{8}\Rightarrow\frac{-37}{8}$

And, $5x-4=5\times\left(\frac{-1}{3}\right)-4$

$=\frac{-5}{8}-4$

$=\frac{-5-32}{8}\Rightarrow\frac{-37}{8}$

$\therefore$ LHS = RHS.

(c) $19x+10=17x+22$

$19x-17x=22-10$

$2x=12$

$x=\frac{12}{2}$

$x=6$

Hence, $(x=6)$ is the solution of the given equation.

Check : $19x+10=19\times 6+10$

$=114+10$

$=124$

And, $17x+22=17\times 6+22$

$=102+22$

$=124$

$\therefore$ LHS = RHS.

(d) $0.35y-2=0.1y+1$

$0.35y-0.1y=1+2$

$0.25y=3$

$y=\frac{3}{0.25}\times\frac{4}{4}$

$y=12$

Hence, $(y=12)$ is the solution of the given equation.

Check : $0.35y-2=0.35\times 12-2$

$=4.2-2\Rightarrow 2.2$

And, $0.1y+1=0.1\times 12+1$

$=1.2+1\Rightarrow 2.2$

$\therefore$ LHS = RHS

2. (a) $\frac{5x+2}{x+4}=3$

$5x+2=3\times(x+4)$

$5x+2=3x+12$

$5x-3x=12-2$

$2x=10$

$x=\frac{10}{2}\Rightarrow 5$

Hence, $(x=5)$ is the solution of the given equation.

Check : $\frac{5x+2}{x+4}=\frac{5\times 5+2}{5+4}$

$=\frac{25+2}{9}$

$=\frac{27}{9}\Rightarrow 3$

$\therefore$ LHS = RHS.

(b) $8x + \frac{3}{4} = 3x + 7$

$8x - 3x = 7 - \frac{3}{4}$

$5x = \frac{28-3}{4}$

$5x = \frac{25}{4}$

$x = \frac{25}{4 \times 5}$

$x = \frac{5}{4}$

Check : $8x + \frac{3}{4} = 8 \times \frac{5}{4} + \frac{3}{4}$

$= \frac{40}{4} + \frac{3}{4}$

$= \frac{40+3}{4} \Rightarrow \frac{43}{4}$

And, $3x + 7 = 3 \times \frac{5}{4} + 7$

$= \frac{15}{4} + 7$

$= \frac{15+28}{4} \Rightarrow \frac{43}{4}$

$\therefore$ LHS = RHS.

Hence, $\left(x = \frac{5}{4}\right)$ is the solution of the given equation.

(c) $7x + \frac{3}{4} = \frac{3}{2}x + 7$

$7x - \frac{3}{2}x = 7 - \frac{3}{4}$

$\frac{14x - 3x}{2} = \frac{28-3}{4}$

$\frac{11}{2}x = \frac{25}{4}$

$x = \frac{25}{4} \times \frac{2}{11}$

$x = \frac{25}{22}$

Check : $7x + \frac{3}{4} = 7 \times \frac{25}{22} + \frac{3}{4} = \frac{175}{22} + \frac{3}{4}$

$= \frac{350+33}{44} \Rightarrow \frac{383}{44}$

And, $\frac{3}{2}x \times 7 = \frac{3}{2} \times \frac{25}{22} + 7$

$= \frac{75}{44} + 7$

$= \frac{75+308}{44} \Rightarrow \frac{383}{44}$

$\therefore$ LHS = RHS

Hence, $\left(x = \frac{25}{22}\right)$ is the solution of the given equation.

(d) $\frac{1}{2}x + 7x - 6 = 7x + \frac{1}{4}$

$\frac{1}{2}x + 7x - 7x = \frac{1}{4} + 6$

$\frac{1}{2}x = \frac{1+24}{4}$

$x = \frac{25}{4} \times 2$

$x = \frac{25}{2}$

Hence, $\left(x = \frac{25}{2}\right)$ is the solution of the given equation.

Check : $\frac{1}{2}x + 7x - 6 = \frac{1}{2} \times \frac{25}{2} + 7 \times \frac{25}{2} - 6$

$= \frac{25}{4} + \frac{175}{2} - 6$

$= \frac{25 + 350 - 24}{4} = \frac{351}{4}$

And, $7x + \frac{1}{4} = 7 \times \frac{25}{2} + \frac{1}{4} = \frac{175}{2} + \frac{1}{4}$

$= \frac{350+1}{4} \Rightarrow \frac{351}{4}$

(e) $\frac{5x-1}{2}-\frac{5x-2}{3}=1$

$\frac{3(5x-1)-2(5x-2)}{6}=1$

$15x-3-10x+4=61\times 6$

$5x+1=6$

$5x=6-1$

$x=\frac{5}{5}=1$

$x=1$

Check : $\frac{5x-1}{2}-\frac{5x-2}{3}$

$=\frac{5\times 1-1}{2}-\frac{5\times 1-2}{3}$

$=\frac{5-1}{2}-\frac{5-2}{3}$

$=\frac{4}{2}-\frac{3}{3}$

$=2-1\Rightarrow 1$

$\therefore$ LHS = RHS.

Hence, $(x=1)$ is the solution of the given equation.

(f) $\frac{5}{4}(7x-3)-\left(4x-\frac{1-x}{2}\right)=x+\frac{3}{2}$

$\frac{35}{4}x-\frac{15}{4}-4x+\frac{1-x}{2}=x+\frac{3}{2}$

$\frac{35}{4}x-\frac{15}{4}+\frac{1-x}{2}-\frac{3}{2}=x+4x$

$\frac{35x-15}{4}+\frac{1-x-3}{2}=5x$

$\frac{35x-15+2-2x-6}{4}=5x$

$33x-19=5x\times 4$

$33x-19=20x$

$33x-20x=19$

$13x=19$

$x=\frac{19}{13}$

Hence, $\left(x=\frac{19}{13}\right)$ is the solution of the given equation.

Check : $\frac{5}{4}(7x-3)-\left(4x-\frac{1-x}{2}\right)$

$=\frac{5}{4}\left(7\times\frac{19}{13}-3\right)-\left(4\times\frac{19}{13}-\frac{\left(1-\frac{19}{13}\right)}{2}\right)$

$=\frac{5}{4}\left(\frac{133-39}{13}\right)-\left[\frac{76}{13}-\frac{1}{2}\left(\frac{13-19}{13}\right)\right]$

$=\frac{5}{4}\times\frac{94}{13}-\left(\frac{76}{13}+\frac{6}{26}\right)$

$$= \frac{235}{26} - \left[\frac{152+6}{26}\right]$$

$$= \frac{235}{26} - \frac{158}{26} \Rightarrow \frac{77}{26}$$

And, $x + \frac{3}{2} = \frac{19}{13} + \frac{3}{2}$

$$= \frac{38+39}{26} \Rightarrow \frac{77}{26}$$

$\because$ LHS = RHS.

(g) $\frac{2}{5}(x-10) = 2x + 3$

$$2(x-10) = 5(2x+3)$$

$$2x - 20 = 10x + 15$$

$$2x - 10x = 15 + 20$$

$$-8x = 35$$

$$x = \frac{35}{-8}$$

Hence, $\left(x = \frac{-35}{8}\right)$ is the solution of the given equation.

Check : $\frac{2}{5}(x-10) = \frac{2}{5}\left(\frac{-35}{8} - 10\right)$

$$= \frac{2}{5}\left(\frac{-35-80}{8}\right)$$

$$= \frac{2}{5 \times 8} \times (-115)$$

$$= \frac{-23}{4}$$

And, $2x + 3 = 2 \times \left(\frac{-35}{8} + 3\right)$

$$= \frac{-35}{4} + 3$$

$$= \frac{-35+12}{4} \Rightarrow \frac{-23}{4}$$

$\therefore$ LHS = RHS.

(h) $\left(\frac{6m-2}{4}\right) + \frac{1}{3}(2m-1) = 4m$

$$\frac{3(6m-2) + 4(2m-1)}{12} = 4m$$

$$18m - 68m - 4 = 4m \times 12$$

$$26m - 10 = 48m$$

$$26m - 48m = 10$$

$$26m - 48m = 10$$

$$-22m = 10$$

$$m = \frac{10}{-22}$$

$$m = \frac{-5}{11}$$

Check : $\left(\frac{6m-2}{4}\right)+\frac{1}{3}(2m-1)=\left(\frac{6\times\frac{-5}{11}-2}{4}\right)+\frac{1}{3}\left(2\times\frac{-5}{11}-1\right)$

$$=\left(\frac{-30-22}{11\times 4}\right)+\frac{1}{3}\left(\frac{-10-11}{11}\right)$$

$$=\frac{-52}{44}-\frac{21}{33}$$

$$=\frac{-13}{11}-\frac{7}{11}$$

$$=\frac{-13-7}{11}\Rightarrow\frac{-20}{11}$$

And, $4m=4\times\frac{-5}{11}=\frac{-20}{11}$

$\therefore$ LHS = RHS.

Hence, $\left(x=\frac{-5}{11}\right)$ is the solution of the given equation.

3. Solve :

(a) $6(3u-1)+3(2u+3)=1-7u$

$$18u-6+6u+9=1-7u$$
$$24u+3=1-7u$$
$$24u+7u=1-3$$
$$31u=-2$$
$$u=\frac{-2}{31}$$

Hence, $\left(u=\frac{-2}{31}\right)$ is the solution of the given equation.

(b) $2-3(3x+1)=2(7-6x)$

$$2-9x-3=14-12x$$
$$-9x-1=14-12x$$
$$-9x+12x=14+1$$
$$3x=15$$
$$x=\frac{15}{3}$$
$$x=5$$

Hence, $(x=5)$ is the solution of the given equation.

(c) $2(3x+2)+\frac{1}{4}=5x-\frac{2}{3}$

$$6x+4+\frac{1}{4}=5x$$
$$6x-5x=-4-\frac{1}{4}$$
$$x=\frac{-16-1}{4}$$
$$x=\frac{-17}{4}$$

Hence, $\left(x=\frac{-17}{4}\right)$ is the solution of the given equation.

(d) $\dfrac{9x-1}{3x+1}=\dfrac{3x-5}{x+6}$

$$(9x-1)(x+6)=(3x-5)(3x+1)$$
$$9x^2-x+54x-6=9x^2+3x-15x-5$$
$$9x^2+53x-6=9x^2-12x-5$$
$$53x+12x=-5+6$$
$$65x=1$$
$$x=\frac{1}{65}$$

Hence, $\left(x=\dfrac{1}{65}\right)$ is the solution of the given equation.

(e) $\dfrac{5}{7+2x}=\dfrac{3}{2x+1}$

$$5(2x+1)=3(7+2x)$$
$$10x+5=21+6x$$
$$10x-6x=21-5$$
$$4x=16$$
$$x=\frac{16}{4}$$
$$x=4$$

Hence, $(x=4)$ is the solution of the given equation.

(f) $\dfrac{8y-1}{2y+1}=\dfrac{4y-5}{y+2}$

$$(8y-1)(y+2)=(4y-5)(2y+1)$$
$$8y^2+16y-y-2=8y^2+4y-10y-5$$
$$8y^2+15y-2=8y^2-6y-5$$
$$15y+6y=-5+2$$
$$21y=-3$$
$$y=\frac{-3}{21}$$
$$y=\frac{-1}{7}$$

Hence, $\left(y=\dfrac{-1}{7}\right)$ is the solution of the given equation.

(g) $\dfrac{9x-(3+4x)}{2x-(7-5x)}=\dfrac{6}{7}$

$$\frac{9x-3-4x}{2x-7+5x}=\frac{6}{7}$$
$$\frac{5x-3}{7x-7}=\frac{6}{7}$$

$$5x-3=\frac{6}{7}\times(7x-7)$$
$$5x-3=6(x-1)$$
$$5x-3=6x-6$$
$$5x-3=6x-6$$
$$5x-6x=-6+3$$
$$-x=-3$$
$$\therefore \quad x=3$$

Hence, $(x=3)$ is the solution of the given equation.

(h) $x-\left(\frac{2x+8}{3}\right)-\frac{x}{4}+\left(\frac{2-x}{24}\right)+3=0$

$$\frac{12x-4(2x+8)-3x}{12}+\left(\frac{2-x}{24}\right)+3=0$$
$$\frac{9x-8x-32}{12}+\frac{2-x+72}{24}=0$$
$$\frac{x-32}{12}+\frac{74-x}{24}=0$$
$$\frac{2(x-32)+(74-x)}{24}=0$$
$$2x-64+74-x=0\times 24$$
$$x-10=0$$
$$x=10$$

Hence, $(x=10)$ is the solution of the given equation.

4. (a) $0.25p-0.05=0.2p+0.15$

$$0.25p-0.2\ p=0.15+0.05$$
$$0.05p=0.20$$
$$p=\frac{0.20}{0.05}=\frac{20}{5}$$
$$p=4$$

Check : $0.25\ p-0.05=0.25\times4-0.05=1-0.05\Rightarrow 0.95$

And, $\quad 0.2p+0.15=0.2\times4+0.15$

$=0.8+0.15\Rightarrow 0.95$

$\therefore$ LHS = RHS.

Hence, $(p=4)$ is the solution of the given equation.

(b) $\frac{(0.25+x)}{3}=x+\frac{1}{2}$

$$0.25+x=3\left(x+\frac{1}{2}\right)$$
$$0.25+x=3x+\frac{3}{2}$$
$$x-3x=1.5-0.25$$

$$-2x = 1.25$$
$$x = \frac{1.25}{-2}$$
$$x = -\ 0.625$$

Hence, $(x = -\ 0.625)$ is the solution of the given equation.

Check : $\frac{(0.25 + x)}{3} = \frac{[0.25 + (-0.625)]}{3}$

$$= \frac{0.25 - 0.625}{3}$$
$$= \frac{-0.375}{3} \Rightarrow 0.125$$

And, $\quad x + \frac{1}{2} = -0.625 + \frac{1}{2}$

$$= -0.625 + 0.5$$
$$= 0.125$$

$\therefore \quad$ LHS = RHS

(c) $0.5x + \frac{x}{4} = 0.25x + 0.5$

$$0.5x + 0.25x = 0.25x + 0.5$$
$$0.5x + 0.25x - 0.25x = 0.5$$
$$0.5x = 0.5$$
$$x = \frac{0.5}{0.5}$$
$$x = 1$$

Hence, $(x = 1)$ is the solution of the given equation.

(d) $\frac{12x + 5}{2} = \frac{3x + 15}{3}$

$$3(12x + 5) = 2(3x + 15)$$
$$36x + 15 = 6x + 30$$
$$36x - 6x = 30 - 15$$
$$30x = 15$$
$$x = \frac{15}{30}$$
$$\therefore \quad x = \frac{1}{2}$$

Hence, $\left(x = \frac{1}{2}\right)$ is the solution of the given equation.

Check : $\frac{12x + 5}{2} = \frac{12 \times \frac{1}{2} + 5}{2} = \frac{6 + 5}{2} \Rightarrow \frac{11}{2}$

And, $\quad \frac{3x + 15}{3} = \frac{3 \times \frac{1}{2} + 15}{3}$

$$= \frac{1}{2} + 5$$
$$= \frac{1 + 10}{2} \Rightarrow \frac{11}{2}$$

$\therefore \quad$ LHS = RHS.

5. (a) $\frac{5x-3}{2x+6}=\frac{3}{2}$

$$2(5x-3)=3(2x+6)$$
$$10x-6=6x+18$$
$$10x-6x=18+6$$
$$4x=24$$
$$x=\frac{24}{4}$$
$$x=6$$

Hence, $(x=6)$ is the solution of the given equation.

(b) $\frac{5x-3}{2x+1}=\frac{2}{5}$

$$5(5x-3)=2(2x+1)$$
$$25x-15=4x+2$$
$$25x-4x=2+15$$
$$21x=17$$
$$x=\frac{17}{21}$$

Hence, $\left(x=\frac{17}{21}\right)$ is the solution of the given equation.

(c) $\frac{3x-4}{2}=\frac{x+2}{3}$

$$3(3x-4)=2(x+2)$$
$$9x-12=2x+4$$
$$9x-2x=4+12$$
$$7x=16$$
$$x=\frac{16}{7}$$

Hence, $\left(x=\frac{16}{17}\right)$ is the solution of the given equation.

(d) $\frac{9x+1}{3x+5}=2$

$$9x+1=2(3x+5)$$
$$9x+1=6x+10$$
$$9x-6x=10-1$$
$$3x=9$$
$$x=\frac{9}{3}$$
$$x=3$$

Hence, $(x=3)$ is the solution of the given equation.

(e) $\frac{9y-1}{3y+2}=\frac{3y-5}{y+6}$

$$(9y-1)(y+6)=(3y-5)(3y+2)$$
$$9y^2+54y-y-6=9y^2+6y-15y-10$$
$$9y^2+53y-6=9y^2-9y-10$$
$$53y+9y=-10+6$$
$$62y=-4$$
$$y=\frac{-4}{62}$$
$$y=\frac{-2}{31}$$

Hence, $\left(y=\frac{-2}{31}\right)$ is the solution of the given equation.

(f) $1\frac{2}{3}x-\frac{x-1}{4}=\frac{x-3}{5}$

$$\frac{5}{3}x-\frac{x-1}{4}-\frac{x-3}{5}=0$$

$$\frac{20\times 5x-15(x-1)-12(x-3)}{60}=0$$

$$100x-15x+15-12x+36=0\times 60$$

$$100x-27x+51=0$$

$$73x=-51$$

$$x=\frac{-51}{73}$$

Hence, $\left(x=\frac{-51}{73}\right)$ is the solution of the given equation.

Exercise 2.2

1. Let, the required number be x.

Thus, $\frac{1}{2}x+x=72$

$$\frac{x+2x}{2}=72$$

$$3x=72\times 2$$

$$x=\frac{72\times 2}{3}$$

$$x=24\times 2\Rightarrow 48$$

Hence, the required number is 48.

2. Let, the required number be x.

Thus, $\frac{3}{4}$ of $x+3=\frac{4}{5}$ of x

$\therefore \quad \frac{4}{5}x-\frac{3}{4}x=3$

$$\frac{16x-15x}{20}=3$$

$$x=3\times 20$$

$$x=60$$

Hence, the required number is 60.

3. Let, the required numbers are $(x-1), x$ and $(x+1)$

$\because$ The sum of three numbers $=36$

$\therefore \quad (x-1)+x+(x+1)=36$

$$x-1+x+x+1=36$$

$$3x=36$$

$$x=\frac{36}{3}\Rightarrow 12$$

Thus, $(x-1)=12-1\Rightarrow 11$

And, $(x+1)=12+1\Rightarrow 13$

Hence, the required the consecutive numbers are 11, 12 and 13 respectively.

4. Given, the ratio of two integers = 2 : 3

Let, the integers are $2x$ and $3x$.

$\because$ Their sum $=35$

$\therefore \quad 2x+3x=35$

$$5x=35$$

$$x=\frac{35}{5}\Rightarrow 7$$

Thus, $2x=2\times 7\Rightarrow 14$

And, $3x=3\times 7\Rightarrow 21$

Hence, the required integers are 14 and 21.

5. Let, the smaller number be x.

And, the greater number be y. ...(1)

Thus, $x + y = 8$

And, $x + 22 = 5y$ ...(2)

By equation (1) and (2) :

$$5y - 22 = 8 - y$$
$$5y + y = 8 + 22$$
$$6y = 30$$
$$y = \frac{30}{6} = 5$$

Put, $(y = 5)$ in equation (1) :

$$2x + 5 = 8$$
$$x = 8 - 5$$
$$x = 3$$

Hence, the smaller number is 3 and the greater number is 5.

6. Let, the digit at unit place $= x$

And, the digit at tens place = y

Thus, the required number $= 10y + x$

By condition, $x = \frac{2}{3}$ of y

$$3x = 2y \quad \text{...(1)}$$

$\because$ Sum of digits = 10

$\therefore$ $x + y = 10$ ...(2)

$$\frac{2}{3}y + y = 10 \quad \left\{\because x = \frac{2}{3}y\right\}$$
$$\frac{2y + 3y}{3} = 10$$
$$5y = 3 \times 10$$
$$y = \frac{30}{5} \Rightarrow 6$$

Put, $(y = 6)$ in equation (1)

$$3x = 2 \times 6$$
$$x = \frac{12}{3} \Rightarrow 4$$

Hence, the required number $= 10y + x$

$= 10 \times 6 + 4 \Rightarrow 4$

7. Let, the digit at unit place $= x$

And, the digit at tens place = y

Thus, the required number $= 10y + x$

By conditions, $x + y = 7$ ...(1)

And, $(10y + x) - 45 = (10x + y)$

$$10y + x - 45 = 10x + y$$

$$10y - y + x - 10x = 45$$
$$9y - 9x = 45$$
$$9(y - x) = 45$$
$$y - x = \frac{45}{9}$$
$$y - x = 5 \quad \ldots(2)$$

Adding equation (1) and (2) :

$$x + y = 7$$
$$y - x = 5$$
$$2y = 12$$
$$y = \frac{12}{2} \Rightarrow 6$$

Put, $(y = 6)$ in equation (1) :

$$x + y = 7$$
$$x = 7 - 6$$
$$x = 1$$

Hence, the required number $= 10y + x$

$= 10 \times 6 + 1 \Rightarrow 61.$

8. Let the denominator of the fraction be x.

Thus, the numerator of the fraction $= x - 3$

$\therefore$ The required fraction $= \frac{x-3}{x}$

By the condition, $\frac{x-3+3}{x+3} = \frac{2}{3}$

$$\frac{x}{x+3} = \frac{2}{3}$$
$$3x = 2(x + 3)$$
$$3x = 2x + 6$$
$$3x - 2x = 6$$
$$x = 6$$

Hence, the required fraction $= \frac{x-3}{x} \Rightarrow \frac{6-3}{6} = \frac{3}{6}.$

9. Let, the present age of Mr. Mumar $= x$ years

And, the present age of his son = y years

Thus, $x + y = 48 \quad \ldots(1)$

By condition, $(x + 2) = 3 \times (y + 2)$

$$x + 2 = 3y + 6$$
$$x = 3y + 6 - 2$$
$$x = 3y + 4 \quad \ldots(2)$$

By equation (1) and (2) :

$$3y + 4 + y = 48$$

$$4y = 48 - 4$$

$$4y = 44$$
$$y = \frac{44}{4} = 11$$
$$y = 11$$
Put $(y = 11)$ in equation (1)
$$x + 11 = 48$$
$$x = 48 - 11$$
$$x = 37$$
Hence, the present of Mr. Mumbar and his son are 37 years and 11 years respectively.

10. Let, the present age of father $= x$ years.
And, the present age of son = y years.
Thus, $x + y = 53$ ...(1)
By condition, $(x-4) = 4 \times (y-4)$
$$x - 4 = 4y - 16$$
$$x = 4y - 16 + 4$$
$$x = 4y - 12 \quad \text{...(2)}$$
By equation (1) and (2) :
$$4y - 12 + y = 53$$
$$5y = 53 + 12$$
$$5y = 65$$
$$y = \frac{65}{5}$$
$$y = 13$$
Put, $(y = 13)$ in equation (1) :
$$x + 13 = 53$$
$$x = 53 - 13$$
$$x = 40$$
Hence, the present age of father and son are 40 years and 13 years.

11. Total length of the wire $= 20$ cm
Let, the breadth of the rectangle formed by wire is x cm.
Thus, the length of the rectangle $= (2x - 2)$ cm.
$\because$ Perimeter of rectangle = length of wire
$\therefore$ $2(l + b) = 20$ cm
$$2[2x - 2 + x] = 20$$
$$2[3x - 2] = 20$$
$$6x - 4 = 20$$
$$6x = 20 + 4$$
$$6x = 24$$
$$x = \frac{24}{6} \Rightarrow 4$$
$\therefore$ Breadth of rectangle $= 4$ cm
And, length of rectangle $= (2 \times 4 - 2)$ cm = 6 cm.

Hence, the length and the breadth of the rectangle are 6 cm and 4 cm respectively.

12. Let, the numerator of the fraction $= x$

Thus, the denominator of the fraction $= 4x - 1$

$\therefore$ The required fraction $= \dfrac{x}{4x-1}$

By condition, $\dfrac{x+1}{4x-1+1} = \dfrac{3}{8}$

$$\frac{x+1}{4x} = \frac{3}{8}$$

$$\frac{x+1}{4x} = \frac{3}{8}$$

$$8 \times (x+1) = 3 \times 4x$$

$$8x + 8 = 12x$$

$$8 = 12x - 8x$$

$$8 = 4x$$

$\therefore$ $x = \dfrac{8}{4} \Rightarrow 2$

Hence, the required fraction $= \dfrac{x}{4x-1} = \dfrac{2}{4\times 2-1} \Rightarrow \dfrac{2}{7}$.

13. Sum of money $=$ ₹ 190

Let, X got money $=$ ₹ x

Y got money $=$ ₹ y

And, z got money $=$ ₹ z

$\because$ y got ₹ 15 more than x.

$\therefore$ $y = x +$ ₹ 15 ...(1)

$\because$ z got ₹ 19 more then y.

$\therefore$ $z = y +$ ₹ 19 ...(2)

By equation (1) :

$z = (x +$ ₹ $15) +$ ₹ 19

$= x +$ ₹ $(15 + 19)$

$z = x +$ ₹ 34 ...(3)

$\because$ sum of ₹ 190 is divided between x, y and z.

$\therefore$ $x + y + z =$ ₹ 190

$x + (x +$ ₹ $15) + (x +$ ₹ $34) =$ ₹ 190

$x + x + x +$ ₹ 15 + ₹ 34 = ₹ 190

$3x +$ ₹ 49 = ₹ 190

$3x =$ ₹ 141

$x =$ ₹ $\dfrac{141}{3} \Rightarrow$ ₹ 47

Thus, $y = x +$ ₹ 15 = ₹ 47 + ₹ 15 $\Rightarrow$ ₹ 62

And, $z = y +$ ₹ 19 = ₹ 62 + ₹ 19 $\Rightarrow$ ₹ 81

Hence, x got ₹ got ₹ 62 and z got ₹ 81.

14. Let, the total distance covered by man is x km.

Thus, distance covered by car $= \frac{2}{5}x$

distance covered by bus $= \frac{3}{10}x$

distance covered by auto-ricksas $= \frac{1}{5}x$

And, distance covered by man on foot $= 5$ km.

$$\therefore \quad \frac{2}{5}x + \frac{3}{10}x + \frac{1}{5}x + 5 \text{ km} = x$$

$$x - \frac{2}{5}x - \frac{3}{10}x - \frac{1}{5}x = 5 \text{ km}$$

$$\frac{10x - 4x - 3x - 2x}{10} = 5 \text{ km}$$

$$10x - 9x = 10 \times 5 \text{ km}$$

$$x = 50 \text{ km}$$

Hence, the total distance covered by man is 50 km.

15. Let, the first part of money be x.

And, the second part of money = y

$\therefore \quad x + y =$ ₹ 200 ...(1)

By condition, $\frac{1}{3}x = \frac{1}{2}y$

$$2x = 3y$$

$$x = \frac{3}{2}y$$

By equation (1) and (2) :

$$\frac{3}{2}y + y = ₹\ 200$$

$$\frac{3y + 2y}{2} = ₹\ 200$$

$$5y = ₹\ 200 \times 2$$

$$y = ₹\ \frac{400}{5} \Rightarrow ₹\ 80$$

Put, $(y = 80)$ is equation (1)

$x +$ ₹ 80 = ₹ 200

$x =$ ₹ 200 – ₹ 80

$x =$ ₹ 120

Hence, ₹ 200 devided into ₹ 120 and ₹ 80 respectively.

16. Let, the number of 50 paise coins $= x$

Thus, the number of 25 paise coins $= 2x$

$\because$ Nitin has money = ₹ 34

or (34×100) paise $= 3400$ paise

$$x \times 50 \text{ p} + 2x + 25 \text{ p} = 3400 \text{ p}$$
$$50x + 50x = 3400$$
$$100x = 3400$$
$$x = \frac{3400}{100} \Rightarrow 34$$

And, $2x = 2 \times 34 \Rightarrow 68$

Hence, Nitin has 34 coins of 50 paise and 68 coins of 25 paise.

17. Let, the cost of a pencil = ₹ x

Thus, the cost of a pen = ₹ $(x+3)$

∴ Cost of 10 pencils $= 10 \times$ ₹ $x \Rightarrow$ ₹ $10x$

And, the cost of 5 pens $= 5 \times$ ₹ $(x+3) \Rightarrow$ ₹ $(5x+15)$

Given the cost of 5 pens and 10 pencils = ₹ 30

∴ ₹ $10x +$ ₹ $(5x+15) =$ ₹ 30

$$10x + 5x + 15 = 30$$
$$15x = 30 - 15$$
$$15x = 15$$
$$x = \frac{15}{15} \Rightarrow 1$$

Hence, the price of each pencil is ₹ 1.

And, the price of each pen is ₹ $(1+3) \Rightarrow$ ₹ 4.

18. Let, the perso bought x pens and y pencils.

∵ Cost of each pen = ₹ 7

∴ Cost of x pens = ₹ $7 \times x \Rightarrow$ ₹ $7x$

∵ Cost of each pencil = ₹ 3

∴ Cost of y pencils = ₹ $3 \times y \Rightarrow$ ₹ $3y$

Thus, $x + y = 108$ ...(1)

And, ₹ $7x +$ ₹ $3y =$ ₹ 564

$7x + 3y = 564$ ...(2)

Multiply equation (1) by (3) :

$3 \times (x + y) = 3 \times 108$

$3x + 3y = 324$ ...(3)

Subtracting equation (3) from equation (2) :

$$7x + 3y = 564$$
$$3x + 3y = 324$$
$$- \quad - \quad -$$
$$4x = 240$$
$$x = \frac{240}{4} \Rightarrow 60$$

Put, $(x = 60)$ in equation (1) :

$$60 + y = 108$$
$$y = 108 - 60$$
$$y = 48$$

Hence, the numbdr of pens and the number of pencils are 60 and 48 respectively.

19. Let, the cost of an ice-cream be x.

Thus, the cost of a chocolate $= 2x$

∵ Cost of 2 choclates bars and 1 ice-cream = ₹ 11.50

∴ $2 \times 2x + 1 \times x =$ ₹ 11.50

$4x + x =$ ₹ 11.50

$5x =$ ₹ 11.50

$x =$ ₹ $\frac{11.50}{5} \Rightarrow$ ₹ 2.30

So, the cost of one chocolate bar $= 2x = 2 \times$ ₹ 2.30

And, the cost of three ice-creams $= 3 \times$ ₹ 2.30 = ₹ 6.90

Hence, the cost of one chocolate bar and thre ice creams is

(₹ 4.60 + ₹ 6.90) $\Rightarrow$ 11.50

20. Total number of days engaged by a labourer = 30

Money received by laboure for working a day = ₹ 60

And, money fined for each day he absent = ₹ 10

Let, the labourer absent x days in the month.

∴ His total income for one month $= (30 - x) \times$ ₹ 60 – $x \times$ ₹ 10

= ₹ $(1800 - 60x - 10x)$

= ₹ $(1800 - 70x)$

∵ He received ₹ 1450 the at end of the month

∴ ₹ $(1800 - 70x) =$ ₹ 1450

$1800 - 70x = 1450$

$1800 - 1450 = 70x$

$350 = 70x$

∴ $x = \frac{350}{70} \Rightarrow 5$

Hence, the laboured absent for 5 days.

21. Percentange of pure silver in old crown = 25%

And, percentage of pure silver in new crown = 40%

∵ Total weight of old crown = 8 kg

∴ Weight of pure silver in old crown = 25% of 8 kg

$= \frac{25}{100} \times 8 \text{ kg}$

$= \frac{8}{4}$ kg $\Rightarrow$ 2 kg

Thus, quantity of other metals in old crown = 8 kg – 2 kg = 6 kg

∵ Quantity of other metals is same in old and new crown.

∴ Quantity of other metals in new crown = 6 kg

Let, total weight of the new crown be x kg.

And, percentage of other metals in new crown $= (100 - 40)\% = 60\%$

∴ 60% of x = 6 kg

$\frac{60}{100} \times x = 6 \text{ kg}$

$$x = \frac{6 \times 100}{60} \text{ kg} \Rightarrow 10 \text{ kg}$$

Thus, quantity of silver in new crow = 10 kg – 6 kg = 4 kg

And, quantity of silver in old crown = 2 kg

$\therefore$ Quantity of silver is added to make new crown = 4 kg – 2 kg = 2 kg.

Hence, 2 kg of silver should be added to the old melted crown to make a new crown.

22. Let, the distance between two given be x km.

Time taken by the streamer goes down stream = 8 hours

And, time taken by the streamer goes up stream = 9 hours

Let, the speed of the streamer in still water = y km/hr

And, speed of the stream = 2 km/hr (given)

$\therefore$ Down stream speed $= \frac{\text{Distance}}{\text{Time}}$

$$(y+2) \text{ km/hr} = \frac{x \text{ km}}{8 \text{ hr}}$$

$$8 \times (y+2) = x \qquad \ldots(1)$$

And, up stream speed $= \frac{\text{Distance}}{\text{Time}}$

$$(y-2) \text{ km/hr} = \frac{x \text{ km}}{9 \text{ hr}}$$

$$9 \times (y-2) = x \qquad \ldots(2)$$

From equation (1) and (2) :

$$9 \times (y-2) = 8 \times (y+2)$$

$$9y - 18 = 8y + 16$$

$$9y - 8y = 16 + 18$$

$$y = 34$$

Hence, the speed of the streamer in still water is 34 km/hr.

Multiple Choice Questions

1. (b) **2.** (c) **3.** (c) **4.** (c) **5.** (b) **6.** (a)
7. (b) **8.** (b) **9.** (c) **10.** (d) **11.** (a)

Brain Teaser

1. False, **2.** True, **3.** False, **4.** False, **5.** True.

Higher Order Thinking Skills

1. Let, the length the base of a triangle be x.

Thus, the altitude of the triangle $= \frac{3}{5}x$

$\therefore$ The area of the triangle $= \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times x \times \frac{3}{5} x$$
$$= \frac{3}{10} x^2$$

Now, new base of the triangle $= x + 4$ cm

And, new altitude of the triangle $= \frac{3}{5} x - 2$ cm

Thus, the new area of the triangle $= \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times (x+4) \times \left(\frac{3}{5} x - 2\right)$$
$$= \frac{1}{2} \times \left[\frac{3}{5} x^2 + \frac{12}{5} x - 2x - 8\right]$$
$$= \frac{3}{10} x^2 + \frac{12}{10} x - x - 4$$
$$= \frac{3}{10} x^2 + \frac{(12x - 10x)}{10} - 4$$
$$= \frac{3}{10} x^2 + \frac{2}{10} x - 4$$
$$= \frac{3}{10} x^2 + \frac{1}{5} x - 4$$

$\because$ Area of the triangle is same.

$$\therefore \quad \frac{3}{10} x^2 = \frac{3}{10} x^2 + \frac{1}{5} x - 4$$
$$\frac{3}{10} x^2 - \frac{3}{10} x^2 = \frac{1}{5} x - 4$$
$$0 = \frac{1}{5} x - 4$$
$$\frac{1}{5} x = 4$$
$$x = 4 \times 5 \Rightarrow 20$$
$$x = 20 \text{ cm}$$

Hence, the base of the triangle is 20 cm.

And, the altitude of the triangle $= \frac{3}{5} \times 20$ cm = 12 cm.

2. Let, the value of first prize be ₹ x.

And, the value of third prize be ₹ y.

Thus, the value of second prize = ₹ $\frac{3}{4} x$ and ₹ $\frac{4}{3} y$

$$\therefore \quad \frac{3}{4} x = \frac{4}{3} y$$
$$3 \times 3 \times x = 4 \times 4 \times y$$
$$9x = 16y \quad \ldots(1)$$

$\because$ Total value of three prizes = ₹ 222

$\therefore \quad x+\frac{3}{4}x+y=₹\ 222$

$x+\frac{3}{4}x+\frac{9}{16}x=₹\ 222 \qquad \left\{\because y=\frac{9}{10}x\right\}$

$\frac{16x+12x+9x}{16}=₹\ 222$

$\frac{37}{16}x=₹\ 222$

$x=\frac{₹\ 222\times 16}{37}$

$x=₹\ (6\times 16)$

$x=₹\ 96$

Hence, the value of first prize = ₹ 96.

The value of second prize $=\frac{3}{4}\times$ ₹ 96 $\Rightarrow$ ₹ 72.

And, the value of third prize $=\frac{9}{16}\times$ ₹ 96 $\Rightarrow$ ₹ 54.

3. Given, the sum of each row, column and diagonals are equal.

x	42	140	77
133	a	98	y
56	b	c	126
d	119	49	112

$\therefore \quad x+42+140+77=133+a+98+y=56+b+c+126$
$=d+119+49+112=x+133+56+d=42+a+b+119$
$=140+98+c+49=77+y+126+112=x+9+c+112=77+98+b+d$

$\because \quad x+133+56+d=119+49+112+d$

$x+189+d=280+d$

$x=280-189$

$x=91$

Now, the sum of each now, column and diagonal $=91+42+140+7=350$

$\therefore \quad 77+y+126+112=350$

$315+y=350$

$y=350-315$

$y=35$

$133+9+98+35=350$

$266+a=350$

$a=350-266$

$a=84$

$$42+84+b+119=350$$
$$245+b=350$$
$$b=350-245$$
$$b=105$$
$$140+98+c+49=350$$
$$287+c=350$$
$$c=350-287$$
$$c=63$$
$$d+119+49+112=350$$
$$d+280=350$$
$$d=350-280$$
$$d=70$$

Hence, the magic square is :

x = 91	42	140	77
133	a = 84	98	y = 35
56	b = 105	c = 63	126
d = 70	119	49	112

3 Understanding Quadrilaterals

Exercise 3

1. (a) If the diagonals of a quadrilateral bisect each other, it is a **parallelogram**.

(b) If the diagonals of a quadrilateral bisect each other at right angles, the quadrilateral is a **rhombus**.

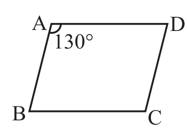
(c) In a square **all** angles are equal and **all** sides are equal. So it is called a **regular** quadrilateral.

(d) In a rhombus, **opposite** angles are equal and **all** sides are equal.

2. One angle of a parallelogram $=130°$

Let, ▭ $ABCD$ is a parallelogram, in which $\angle A=130°$.

In the given parallelogram $ABCD$, $\angle A$ and $\angle B$ are adjacent angles.



Therefore, $\angle A+\angle B=180°$ (co-interior angles)

$$130°+\angle B=180°$$
$$\angle B=180°-130°$$
$$\angle B=50°$$

Now, opposite angles of a parallelogram are equal.

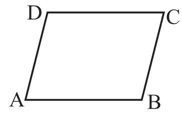
Therefore, $\angle A=\angle C=130°$

and $\angle B=\angle D=50°$

Hence, the measure of the remaining angles of parallelogram are 50°, 130° and 50°.

3. Since, two adjacent angles of a parallelogram are equal. So, the is another name for this parallelogram is rectangle.

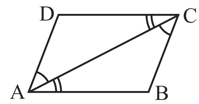
4. The measurements of all the sides are equal. The another name for the figure is Rhombus..



5. The specific name for this figure is square.

6. **Given,**

$ABCD$ is a parallelogram.



$\therefore \quad \angle B = \angle D$

(opposite angles of a parallelogram are equal.)

In ΔABC and ΔADC,

$AD = BC$

$AB = DC$ (opposite sides of a parallelogram)

$\angle ADC = \angle ABC$

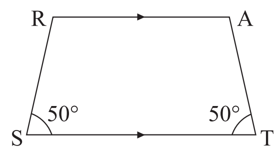
(opposite angles of a parallelogram)

$\therefore \quad \Delta ABC \cong \Delta ADC$ (By SAS congruency)

Hence, AC divides parallelogram $ABCD$ into two congruent triangles.

7. In trapezium $STAR$,

$\angle S$ and $\angle R$ are adjacent angles.



Therefore, $\angle S + \angle R = 180°$

$50° + \angle R = 180°$

$\angle R = 180° - 50°$

$\Rightarrow \quad \angle R = 130°$

Similarly $\angle T + \angle A = 180°$

$50° + \angle A = 180°$

$\angle A = 180° - 50°$

$\Rightarrow \quad \angle A = 130°$

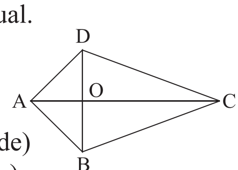
Hence, $\angle R = \angle A$

$= 130°.$

8. In kite $ABCD$,

(a) Yes, because two pairs of adjacent sides are equal.

(b) Yes, because two pairs of adjacent sides are equal.



(c) Yes,

Because $AB = AD$

and $BC = DC$

and $AC = AC$ (Common side)

So, $\Delta ADC \cong \Delta ABC$ (By SSS rule)

(d) Yes,

Because $AB = AD$

and $BC = DC$

and $\angle ADC = \angle ABC$

(One pair of diagonally opposite angles is equal.)

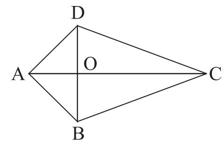
(e) Yes, because diagonal AC is bisect $\angle DAB$.

$\therefore \quad \angle DAO \cong \angle BAO.$

(f) Yes, because diagonal AC is bisect $\angle DCB$

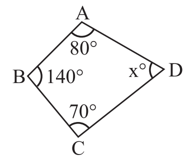
$\therefore \quad \angle DCA \cong \angle BCA.$

9. In kite *ABCD*,



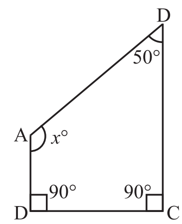
(a) Yes
(b) Yes
(c) Yes
(d) Yes
(e) Yes

10. (a) We know that,



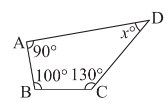
$$\angle A + \angle B + \angle C + \angle D = 360°$$
$$80° + 140° + 70° + \angle D = 360°$$
$$290° + \angle D = 360°$$
$$x = \angle D = 360° - 290° \; (\because \angle D = x)$$

So, $x = 70°$.



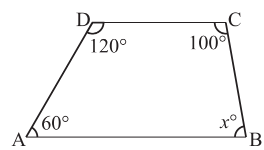
(b) $\because \angle A + \angle B + \angle C + \angle D = 360°$
$$x + 90° + 90° + 50° = 360°$$
$$x + 230° = 360°$$
$$x = 360° - 230°$$

So, $x = 130°$



(c) $\angle A + \angle B + \angle C + \angle D = 360°$
$$90° + 100° + 130° + x = 360°$$
$$320° + x = 360°$$
$$x = 360° - 320°$$

So, $x = 40°$



(d) $\angle A + \angle B + \angle C + \angle D = 360°$
$$60° + x + 100° + 120° = 360°$$
$$x + 280° = 360°$$
$$x = 360° - 280°$$

So, $x = 80°$

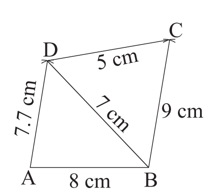
Multiple Choice Questions

1. (b) **2.** (c) **3.** (a)

4 Practical Geometry

Exercise 4.1

1. Steps of construction :



Step 1. First draw a line segment $AB = 8$ cm.

Step 2. With *A* and *B* as centres and radii 7.7 cm and 7 cm respectively. draw two arcs above *AB* to cut each other at *D*. Join *AD* and *BD*.

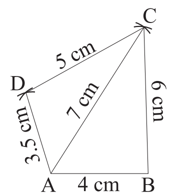
Step 3. Again with *B* and *D* as centres and radii 9 cm and 5 cm respectively, draw two arcs above BD to cut each other at C. Join DC and BC.

Thus, $ABCD$ is the required quadrilateral.

2. Steps of construction :

Step 1. Draw a line segment $AB = 4$ cm.

Step 2. With A and B as centres and radii 7 cm and 6 cm respectively, draw two arcs above AB to cut each other at C. Join AC and BC.



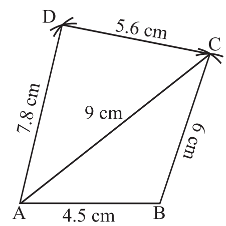
Step 3. Again with C and A as centres and radii 5 cm and 3.5 cm respectively, draw two arcs above AC to cut each other at D. Join AD and DC.

Thus, $ABCD$ is the required quadrilateral.

3. Steps of construction :

Step 1. Draw a lin e$AB = 4.5$ cm.

Step 2. With A and B as centres and radii 9 cm and 6 cm respectively, draw two arcs above AB to cut each other at C. Join AC and BC.



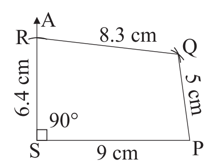
Step 3. Again with A and C as centres and radii 7.8 cm and 5.6 cm respectively, draw two arcs above AC to cut each other at D. Join AD and DC.

Thus, $ABCD$ is the required quadrilaterals.

4. Steps of construction :

Step 1. Draw a line segment $SP = 9$ cm.

Step 2. Construct $\angle PSA = 90°$ with S as centre and radius $SR = 6.4$ cm, cut off SR on SA.



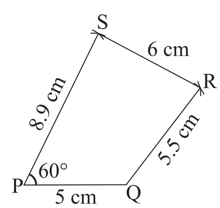
Step 3. With R and P as centres and radii 8.3 cm and 5 cm respectively, draw two arcs to cut each other at Q. Join RQ and PQ.

Thus, $PQRS$ is the required quadrilateral.

5. Steps of construction :

Step 1. Draw a line segment $PQ = 5$ cm.

Step 2. Construct $\angle QPA = 60°$, with P as centre and radius $PS = 8.9$ cm, cut off PS on PA.



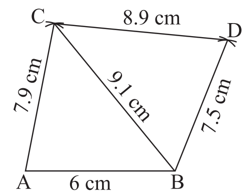
Step 3. With Q and S as centres and radii 5.5 cm and 6 cm respectively, draw two arcs to cut each other at R. Join QR and SR.

Thus, $PQRS$ is the required quadrilateral

6. Steps of construction :

Step 1. Draw a line segment $AB = 6$ cm.

Step 2. With A and B as centres and radii 7.9 cm and 9.1 cm respectively, draw two arcs to cut each other at C. Join AC and BC.



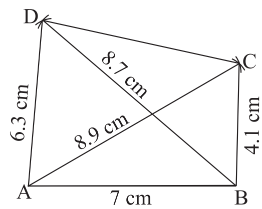
Step 3. Again with B and C as centeres and radii 7.5 cm and 8.9 cm respectively, draw two arcs to cut each other at D. Join BD and CD.

Thus, $ABCD$ is the required quadrilateral.

7. Steps of construction :

Step 1. Draw a line segment $AB = 7$ cm.

Step 2. With A and B as centres and radii 8.9 cm and 4.1 cm respectively, draw two arcs to cut each other at C. Join AC and BC.



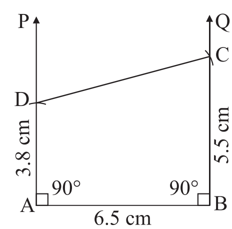
Step 3. Again with A and B as centres and radii 6.3 cm and 8.7 cm respectively, draw two arcs to cut each other at D. Join AD and BD.

Thus, $ABCD$ is the required quadrilateral.

8. Steps of construction :

Step 1. Draw a line segment AB of length 6.5 cm.

Step 2. Construct $\angle BAP = 90°$.



Step 3. With A as centre and radius 3.8 cm, cut off $AD = 3.8$ cm on the ray AP.

Step 4. Construct $\angle ABQ = 90°$, with B as centre and radius 5.5 cm, cut off BC on the ray BQ.

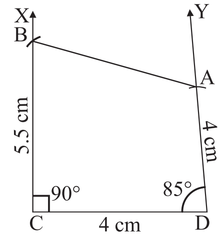
Step 5. Now, join DC.

Thus, $ABCD$ is the required quadrilateral.

9. Steps of construction :

Step 1. Draw a line segment $CD = 4$ cm.

Step 2. Construct $\angle DCB = 90°$, with C as centre and radius $CB = 5.5$ cm, cut off CB on ray CX.



Step 3. Construct $\angle CDY = 85°$, with D as centre and radius $DA = 4$ cm, cut off DA on ray DY .

Step 4. Join AB.

Thus, $ABCD$ is the required quadrilateral.

10. Steps of construction :

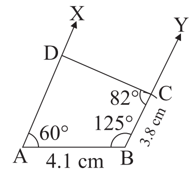
Step 1. Draw a line segment $AB = 4.1$ cm.

Step 2. Construct $\angle BAX = 60°$.

Step 3. Construct $\angle ABY = 125°$, with B as centre and radius $BC = 3.8$ cm, cut off BC on BY.

Step 4. Construct $\angle BCD = 82°$.

Thus, $ABCD$ is the required quadrilateral.



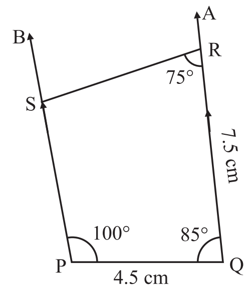
11. Steps of construction :

Step 1. Draw a line segment $PQ = 4.5$ cm.

Step 2. Construct $\angle PQA = 85°$, with Q as centre and radius 7.5 cm, cut off $QR = 7.5$ cm on QA.

Step 3. Construct $\angle QRS = 75°$, and construct $\angle PSR = 100°$.

Thus, $PQRS$ is the required quadrilateral.



12. Steps of construction :

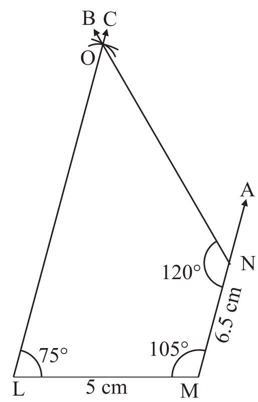
Step 1. Draw a line segment $LM = 5$ cm and at M make an angle $\angle LMA = 105°$ and at L make an angle $\angle MLC = 75°$.

Step 2. With M as centre cut MA with an arc of radius 6.5 cm off and mark the intersecting point as N.

Step 3. At N, draw $\angle MNB = 120°$.

Step 4. Let the two rays LC and NB, cut each other at point O.

Thus, $LMNO$ is the required quadrilateral.



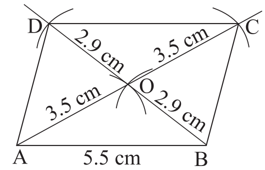
Exercise 4.2

1. In this case, we make use of the property that the diagonals of a parallelogram bisect each other.

Steps of construction :

Step 1. Draw a line segment $AB = 5.5$ cm.

Step 2. With A as centre draw an arc of radius 3.5 cm and with B as centre draw another arc of radius 2.9 cm to cut each other at point O.



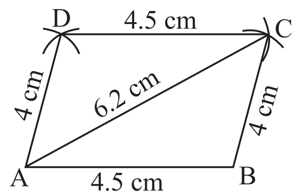
Step 3. Now, join OA and OB.

Step 4. Extend AO to C such that $OC = OA$ and BO to D such that $OD = OB$.

Step 5. Now, join AD, BC and DC.

Thus, $ABCD$ is the required parallelogram.

2. Steps of construction :



Step 1. Draw a line segment $AB = 4.5$ cm.

Step 2. With A as centre draw an arc of radius 6.2 cm.

Step 3. Now, with B as centre draw another arc of radius 4 cm to cut the previous arc at point C.

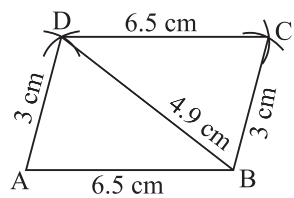
Step 4. Now, join BC and AC.

Step 5. With A as centre draw an arc of radius 4 cm and with C as centre draw an other arc of radius 4.5 cm to cut each other at point D.

Step 6. Now, join CD and AD.

Thus, $ABCD$ is the required parallelogram.

3. Steps of construction :



Step 1. Draw a line segment $AB = 6.5$ cm.

Step 2. With B as centre draw an arc of radius 4.9 cm.

Step 3. Now, with A as centre draw another arc of radius 3 cm to cut the previous arc at point D.

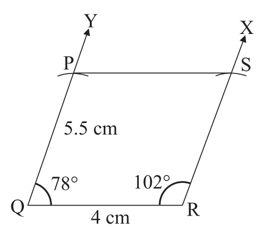
Step 4. Now, join AD and BD.

Step 5. With B as centre draw an arc of radius 3 cm and with D as centre draw an other arc of radius 6.5 cm to cut each other at point C.

Step 6. Now, join CD and BC.

Thus, $ABCD$ is the required parallelogram.

4. If one angle of a parallelogram is 78°, then its adjacent angles will be (180° – 78°). It should be noted that the adjacent angles of a parallelogram are supplementary.



Steps of construction :

Step 1. Draw a line segment $QR = 4$ cm.

Step 2. At Q draw $\angle RQY = 78°$.

Step 3. With Q as centre draw an arc of radius 5.5 cm cutting QY at P.

Step 4. At R draw $\angle QRX = 102°$.

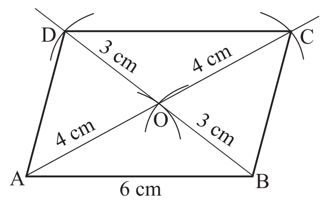
Step 5. With R as centre draw an arc of radius 5.5 cm cutting RX at S.

Step 6. Join PS.

Thus, $PQRS$ is the required parallelogram.

5. In this case, we make use of the property that the diagonals of a parallelogram bisect each other.

Steps of construction :



Step 1. Draw a line segment $AB = 6$ cm.

Step 2. With A as centre draw an arc of radius 4 cm and with B as centre draw another arc of radius 3 cm to cut each other at point O.

Step 3. Now, join OA and OB.

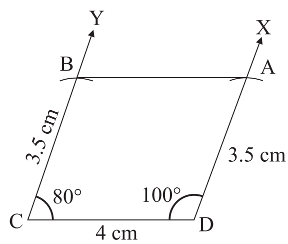
Step 4. Extend AO to C such that $OC = OA$ and BO to D such that $OD = OB$.

Step 5. Now, join AD, BC and DC.

Thus, $ABCD$ is the required parallelogram.

6. If one angle of a parallelogram is 80°, then its adjacent angles will be (180° – 80°) = 100°. It should be noted that the adjacent angles of a parallelogram are supplementary.

Steps of construction :



Step 1. Draw a line segment $CD = 4$ cm.

Step 2. At C as centre draw $\angle DCY = 80°$.

Step 3. With C as centre draw an arc of radius 3.5 cm cutting ray CY at B.

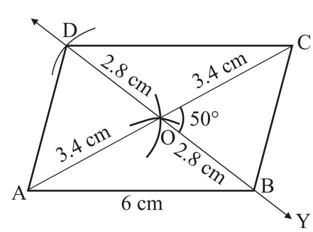
Step 4. At as centre D draw $\angle CDX = 100°$.

Step 5. With D as centre draw an arc of radius 3.5 cm cutting ray DX at A.

Step 6. Join AB.

Thus, $ABCD$ is the required parallelogram.

7. Steps of construction :



Step 1. Draw a line segment $AC = 6.8$ cm.

Step 2. With A as centre draw an arc of radius 3.4 cm cut AC at point O.

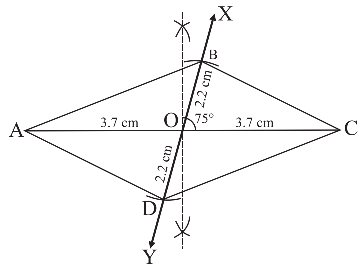
Step 3. At O draw $\angle COY = 50°$.

Step 4. Draw a line XY, at O as centre draw two arcs of radii 2.8 cm both sides of point O, cut XY at B and D.

Step 5. Join AD, AB, BC and DC.

Thus, $ABCD$ is the required parallelogram.

8. Steps of Construction :



Step 1. Draw a line segment $AC = 7.4$ cm. Now, bisect AC and mark it as point O.

Step 2. Draw $\angle COX = 75°$ and produce XO to Y.

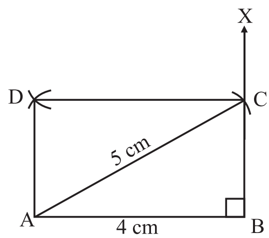
Step 3. Now, cut off $OB = \frac{1}{2}(7.4 \text{ cm})$

$= 3.7$ cm and $OD = \frac{1}{2}$ (7.4 cm) $= 3.7$ cm as show in figure.

Step 4. Now, Join AB, BC, CD and DA.

Thus, $ABCD$ is the required parallelogram.

9. Steps of Construction :



Step 1. Draw a line segment $AB = 4$ cm.

Step 2. Draw a line $BX \perp AB$.

Step 3. With A as centre draw an arc of radius 5 cm, cut BX at point C.
Join AC.

Step 4. With C as centre draw an arc of radius 4 cm shown in Fig.

Step 5. With A as centre draw an arc equal to the radius BC, to cut the previous arc at D.

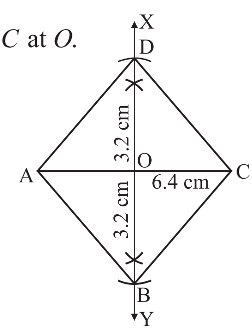
Step 6. Join AD and DC.

Thus, $ABCD$ is the required rectangle.

10. Steps of Construction :

Step 1. Draw a line segment $AC = 6.4$ cm.

Step 2. Draw the right bisector XY of AC cut AC at O.



Step 3. On OX, cut off

$OD = \frac{1}{2} \times$ (6.4 cm) = 3.2 cm.

Similarly, on OY, cut off

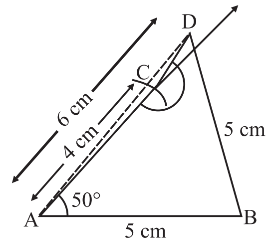
$OB = \frac{1}{2} \times$ (6.4 cm) = 3.2 cm.

Step 4. Join AB, BC, CD and DA.

Thus, $ABCD$ is the required square.

HOTS

1. Yes, $ABCD$ quadrilateral in which at least one of the angle $\angle ACD$ more than 180°.
   So, it is a concave quadrilateral.
   ∴ It is a unique quadrilateral.



2. **Steps of Construction :**

   Step 1. Draw a line segment $LM = 3.5$ cm.

   Step 2. With L as centre and radius $LO = 3.5$ cm draw an arc.

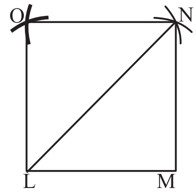
   Step 3. With M as centre and radius $MO = 5$ cm draw an other arc, cutting previou arc at point O.

   Step 4. Join LO and OM.

   Step 5. With O and M as centres and radius 3.5 cm, draw two arcs which intersect each other at point N.

   Step 6. Join ON and MN.

   Thus, $LMNO$ is the required rhombus.



3. Given, $PS = 7.5$ cm, $PQ = 3$ cm and $SQ = 4$ cm

   ∵ $PQ + SQ = 3$ cm + 4 cm

   ⇒ 7 cm and 7.5 cm > 7 cm

   ∴ $PS > PQ + SQ$

   So, the construction of this quadrilateral $PQRS$ is not possible.

---

5 Data handling

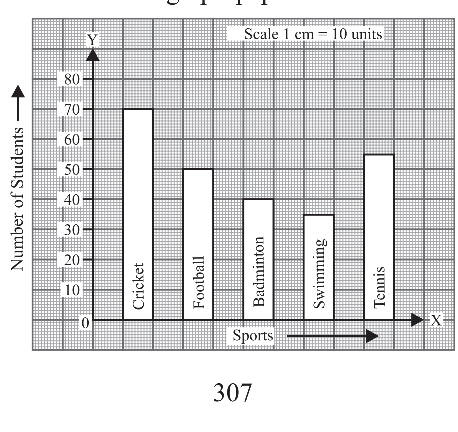
Exercise 5.1

1. **Look at the bar graph and answer the questions that follow :**
   (a) The title given to the bar graph is public expenditure on health and nutrition.
   (b) Brazil has the minimum expenditure on health and nutritions.
   (c) Denmark has ten thousand crores expenditure on health and nutritions.
   (d) The ratio of expenditure on health and nutrition of India and New Zealand = 7 : 11.
2. **Read the graph and answer the following questions :**
   (a) The period covered in the bar graph is 5 years.
   (b) The fees increase from 2018 to 2022 was (₹ 600 – ₹ 400) = ₹ 200.
   (c) The fees was minimum in 2018.
   (d) The maximum fees was ₹ 700 per month.
   (e) No.

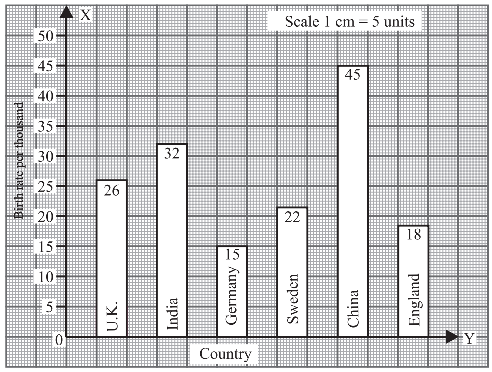
3. **The following bar graph represents the data on crimes in respect of five states in the year 2022 :**
   (a) U.P. had reported the maximum number of crimes.
   (b) Goa had reported the minimum number of crimes.
   (c) The ratio of Maximum no. of crimes to minimum no. of crimes
   $= 4000 : 1000 = 4 : 1$
   (d) The difference between the no. of crimes reported in Bihar and Tamil Nadu $= 3500 - 3000 = 500.$
4. **Read the bar graph carefully and answer the following questions :**
   (a) Mount Everest is the highest peak and its heights is 8800 m.
   (b) The Ratio of highest peak and the next highest peak
   $= 8800 : 8200 = 44 : 41$
   (c) Annapurna is the lowest peak.
   (d) The difference between the heights of the highest and the lowest peaks is
   $= (8800 \text{ m} - 6000 \text{ m}) = 2800 \text{ m}.$
   (e) The height of peaks in descending order
   $= 8800 > 8200 > 8000 > 7500 > 6000.$
   (f) The difference between the heights of Nanda Devi peak and Annapurna peak $= 7500 \text{ m} - 6000 \text{ m} = 1500 \text{ m}.$
5. **Read the graph and answer the following questions :**
   (a) The production of rice in India is 100 million tonnes.
   (b) Bangladesh has the minimum production of rice.
   (c) China has the maximum production of rice.
   (d) Thailand produces 60 million tonnes of rice.
6. **Look at the bar graph and answer the questions that follow :**
   (a) The maximum temperature of the day was 70°F.
   (b) The temperature was recored as 55°F at 6 : 00 pm.
   (c) The temperature was 25°F at 5 : 00 am.
   (d) The difference between the maximum and the minimum temperatures of the day was $(70 - 25)$ °F $= 45$°F.

Exercise 5.2

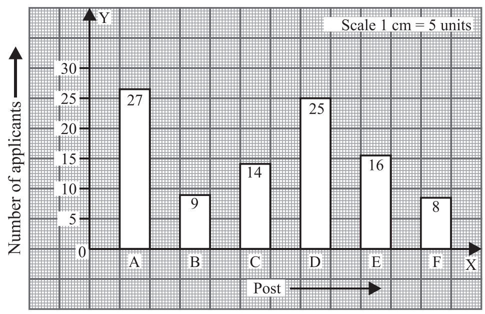
1. To represent this data on a graph paper.



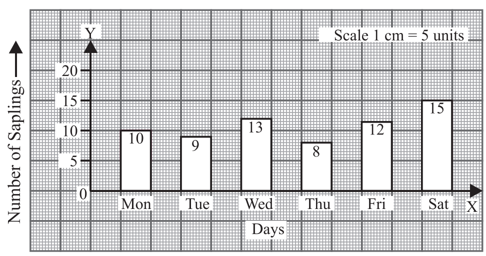
2. To represent this data on a bar graph paper.



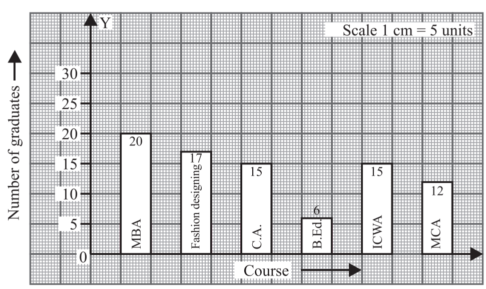
3. To represent this data on a graph paper.



4. To represent this data on a graph paper.

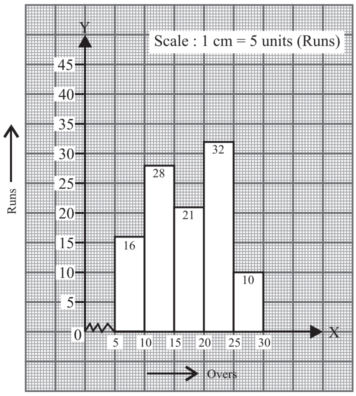


5. To represent this data on a graph paper.

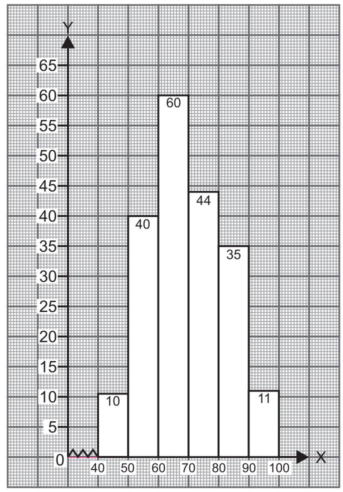


Exercise 5.3

1.



2. **Scale :** 1 cm = 5 students

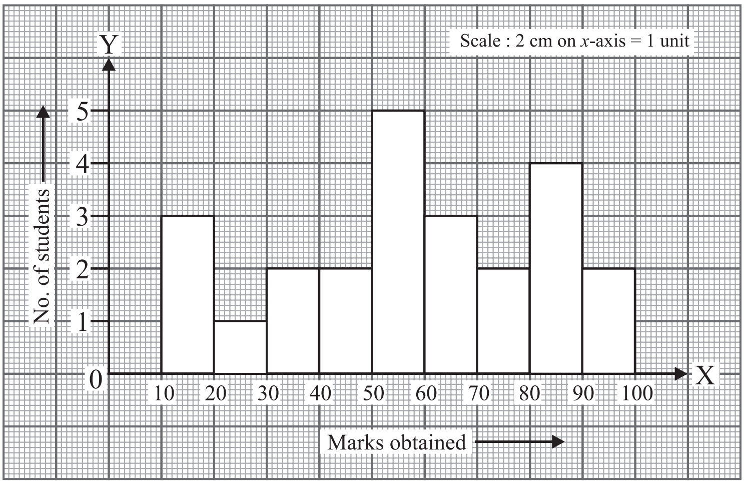


3. (a) There are 6 teachers in the age group of (50 – 55) years.
(b) There are 13 teachers above the age of 49 years.
(c) There are 4 teachers below the age of 35 years.
(d) Age group of (45 – 50) has maximum no. of teachers.
(e) The age group of (30 – 35) years has minimum no. of teachers.

4. (a) The class size of each class interval is 20.
(b) The interval of maximum earnings is (40 – 60).
(c) The interval of minimum earnings is (0 – 20).
(d) $2+5+10=17$ chemists earn less than ₹ 60 per day.

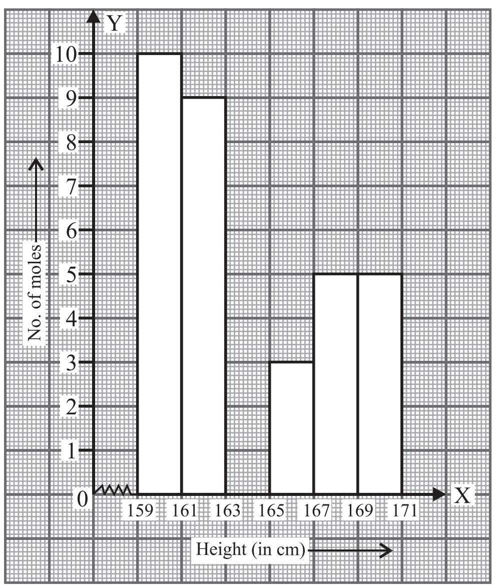
5.

<table>
<tr><th>S.No.</th><th>Class Interval</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>1.</td><td>0-10</td><td>—</td><td>0</td></tr>
<tr><td>2.</td><td>10-20</td><td>|||</td><td>3</td></tr>
<tr><td>3.</td><td>20-30</td><td>|</td><td>1</td></tr>
<tr><td>4.</td><td>30-40</td><td>||</td><td>2</td></tr>
<tr><td>5.</td><td>40-50</td><td>||</td><td>2</td></tr>
<tr><td>6.</td><td>50-60</td><td>~~||||~~</td><td>5</td></tr>
<tr><td>7.</td><td>60-70</td><td>|||</td><td>3</td></tr>
<tr><td>8.</td><td>70-80</td><td>||</td><td>2</td></tr>
<tr><td>9.</td><td>80-90</td><td>||||</td><td>4</td></tr>
<tr><td>10.</td><td>90-100</td><td>||</td><td>2</td></tr>
<tr><td></td><td></td><td>Total</td><td>24</td></tr>
</table>



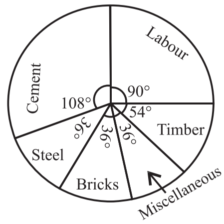
6.

S.No.	Class Interval	Tally Marks	Frequency
1.	159-161	|||| ||||	10
2.	161-163	|||| ||||	9
3.	163-165	—	0
4.	165-167	|||	3
5.	167-169	||||	5
6.	169-171	||||	5
		Total	32



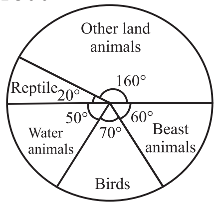
Exercise 5.4

1.



Items	Expenditure	Sector Angle
Cement	30%	$\frac{30}{100} \times 360° = 108°$
Steel	10%	$\frac{10}{100} \times 360° = 36°$
Bricks	10%	$\frac{10}{100} \times 360° = 36°$
Timber	15%	$\frac{15}{100} \times 360° = 54°$
Labour	25%	$\frac{25}{100} \times 360° = 90°$
Miscellaneous	10%	$\frac{10}{100} \times 360° = 36°$
Total	100%	360°

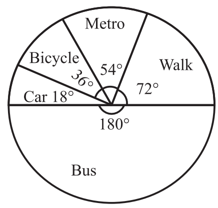
2. Number of Creatures $= 1800$



Creatures in Zoo	No. of Animals	Sector angle
Beast animals	300	$\frac{300}{1800} \times 360° = 60°$
Other land animals	800	$\frac{800}{1800} \times 360° = 160°$
Birds	350	$\frac{350}{1800} \times 360° = 70°$
Water animals	250	$\frac{250}{1800} \times 360° = 50°$

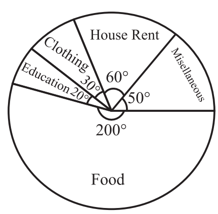
Reptile	100	$\frac{100}{1800} \times 360° = 20°$
Total	1800	360°

3. Total no. of students = 2000



Transport to school	Number of students	Central angle
Walk	400	$\frac{400}{2000} \times 360° = 72°$
Bus	1000	$\frac{1000}{2000} \times 360° = 180°$
Bicycle	200	$\frac{200}{2000} \times 360° = 36°$
Metro	300	$\frac{300}{2000} \times 360° = 54°$
Car	100	$\frac{100}{2000} \times 360° = 18°$
Total	2000	360°

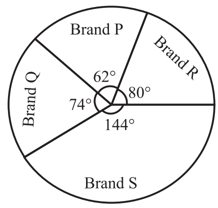
4. Monthly salary = ₹ 7200



Items	Amount Spent (in ₹)	Sector angle
Clothing	600	$\frac{600}{7200} \times 360° = 30°$
Food	4000	$\frac{4000}{7200} \times 360° = 200°$
House rent	1200	$\frac{1200}{7200} \times 360° = 60°$

Education	400	$\frac{400}{7200} \times 360° = 20°$
Miscellaneous	1000	$\frac{1000}{7200} \times 360° = 50°$
Total	7200	360°

5.



Brand	No. of people	Sector angle
Brand *P*	31	$\frac{31}{180} \times 360° = 62°$
Brand *Q*	37	$\frac{37}{180} \times 360° = 74°$
Brand *R*	40	$\frac{40}{180} \times 360° = 80°$
Brand *S*	72	$\frac{72}{180} \times 360° = 144°$
Total	180	360°

6. (a) Let total amount spent be x.

Now, money spent on football = ₹ 900

So, $x \times \frac{20°}{360°} = ₹\ 900$

$x = \frac{900 \times 360°}{20°}$

$x = ₹\ 16200$

So, total amount spent on sports is ₹ 16200.

(b) Sector angle of tennis = 35°

So, money spent on tennis = $\frac{35°}{360°} \times ₹16200 = ₹\ 1575$.

(c) Money spent on hockey = $\frac{100°}{360°} \times ₹\ 16200 = ₹\ 4500$

And, money spent on football = ₹ 900

∴ difference = ₹ (4500 − 900) = ₹ 3600

So, ₹ 3600 less was spent on football than Hockey.

(d) Since sector angle of cricket is greater than all other angles.

So, amount spent on cricket was maximum.

(e) Since, sector angle of football is smallest among all angles.
So, amount spent on football was minimum.

7. (a) Let the market value of total share $= x$

∵ Market value of share of company S (given) = ₹ 8 crores
= ₹8,00,00,000

Central angle of company $S = 80°$

So, $$x \times \frac{80°}{360°} = 8,00,00,000$$

$$x = ₹\ \frac{8,00,00,000 \times 360}{80}$$

x = ₹ 360000000 or 36 crores.

(b) Since, sector angle of company V is greater than all other angles.
So, the company V has the maximum market share.

(c) Total market value (given) = ₹ 72 crores = ₹ 72,00,00,000
Central angle of U = 100°

So, the share of Company $U = \frac{100°}{360°} \times$ ₹ 72,00,00,000
= ₹ 20,00,00,000 or 20 crores.

Exercise 5.5

1. (a) (3, 2) is a simple event.
(b) (4, 4) is not a simple event.
because we cant not draw two fours at a time from {1, 2, 3, 4, 5}.
(c) (3, 3) is not a simple event.
Because we can not draw two threes at a time from {1, 2, 3, 4, 5, 6}.
(d) ∵ 1, 3, 5, 7 and 9 are odd numbers from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
So, it is a simple event.

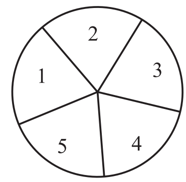
2. (a) Sample space = {blue, green, white, red}
Simple events = (blue), (green), (white), (red).
(b) Sample space = {red, green, blue}
Simple events = (red, green), (red, blue), (green, red), (green, blue), (blue, red), (blue, green).
(c) Sample space = {2, 2, 2, 5}
Simple events = (2), (5).
(d) Sample space = {green, green, green, green, green, green, green, green, blue, blue, blue, blue, blue, blue, blue, blue, blue, blue, red, red, red, red, red, red}
Simple events = (green), (blue), (red).
(e) Simple events = (HHH), (TTT), (HHT), (TTH).
(f) Simple events = (1), (2), (3), (4), (5), (6).
No, (1, 2) is not an event of the sample space {1, 2, 3, 4, 5, 6}

Exercise 5.6

1. Total number of cards in a pack of cards = 52
And, the number of ace of clubs = 1

So, the probability of an ace of clubs $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{1}{52}.$

2. In this given spinner.



$\therefore$ Sample space = {1, 2, 3, 4, 5}

And, Simple events = (1), (2), (3), (4), (5).

Now, the probability of number 4

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{1}{5}.$$

And, the probability of an even number $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{2}{5}.$

3. Sample space of tetrahedron = {green, red, blue, yellow}

And simple events = (green), (red), (blue), (yellow).

Now, the probability of red colour $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{1}{4}.$

4. Number of green balls in a bag = 30

And, number of white balls in bag = 30

$\therefore$ Total number of balls in the bag $= 30 + 30 = 60.$

Now, the probability of drawing a white ball $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}}$

$$= \frac{30}{60} \Rightarrow \frac{1}{2}.$$

5. $\because$ A coin tossed three times.

$\therefore$ Total number of outcomes = $2^3 \Rightarrow 8$.

Sample space = {HHH, HHT, HTH, HTT, TTT, TTH, THT, TTH}

Now, favourable outcome = (HHH) = 1

So, the probability of getting heads all three times

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{1}{8}.$$

6. Total number of slips = 4

Sample space = {7, 8, 9, 10}

Now, probability of getting 7 numbered slip $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{1}{4}.$

$\because$ First slip is replaced before drawing the second slip.

$\therefore$ Total outcomes of second trial = 4 (remain)

Thus, probability of getting 9 numbered slip $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}} = \dfrac{1}{4}.$

So, the probability of obtaining 7 and then 9 = $p_1 \times p_2 = \dfrac{1}{4} \times \dfrac{1}{4} \Rightarrow \dfrac{1}{16}.$

Exercise 5.7

1. Total number of outcomes if two dice are tossed together $= 6 \times 6 \Rightarrow 36$

Simple events of a sum of $9 = (3, 6), (4, 5), (5, 4), (6, 3)$

$\therefore$ Favourable outcomes = 4

So, probability of getting a sum of 9 $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

$$= \frac{4}{36} \Rightarrow \frac{1}{9}.$$

2. Total number of outcomes of a dice tossing twice $= 6 \times 6 \Rightarrow 36$

Simple event of same numbers on both times

= (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ Favourable outcomes = 6

So, the probability of getting the same number both time

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{6}{36} \Rightarrow \frac{1}{6}.$$

3. Total number of outcomes of a dice = 6.

Simple event of getting a number equal to or less than 4 = 1, 2, 3, 4.

$\therefore$ Favourable outcomes = 4.

So, the probability of getting a number equal to or less than 4

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{4}{6} \Rightarrow \frac{2}{3}.$$

4. Total outcomes of two coins tossed = {(HH), (HT), (TH), (TT)}

Simple event of getting at least one tails = (HT), (TH), (TT)

$\therefore$ Probability of getting at least one tails when two coins are tossed

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{3}{4}.$$

5. A bag contain red, green and blue balls.

$\therefore$ Sample space = {(RRR), (RRG), (RRB), (GGG), (GGR), (GGB), (BBB), (BBR), (BBG), (RGB)}

And, successful events of getting same colour balls

= {(RRR), (GGG), (BBB)}.

6. A coin is tossed twice.

$\therefore$ Sample space = {(HH), (HT), (TH), (TT)}

And, successful events of getting two heads or two tails = {(HH), (TT)}

So, it is a compound event.

7. Two coins are tossed.

$\therefore$ Sample space = {(HH), (HT), (TH), (TT)}

And, successful events of getting at least of the heads = {(HH), (HT), (TH)}

So, it is a compound event.

8. If a dice are thrown twice.

$\therefore$ Sample space = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

And, successful events of the sum is less than 6
= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

Multiple Choice Questions

1. (b), 2. (c), 3. (d), 4. (d), 5. (a), 6. (a), 7. (c), 8. (a), 9. (c), 10. (d).

Brain Teaser

Fill in the blanks :

1. Probability of an event cannot be **negative**.
2. Probability of an event always lies between **0** and **1**.
3. Probability of a sure event is always **1**.
4. Probability of an impossible event is **0**.
5. **Sample space** is a collection of all possible outcomes.

6 Square and Square Roots

Exercise 6.1

1. (a) $19^2 = 19 \times 19 = 361$ (b) $25^2 = 25 \times 25 = 625$

(c) $11^2 = 11 \times 11 = 121$ (d) $39^2 = 39 \times 39 = 1521$

(e) $115^2 = 115 \times 115 = 13225$ (f) $103^2 = 103 \times 103 = 10609$

(g) $123^2 = 123 \times 123 = 15129$ (h) $45^2 = 45 \times 45 = 2025$

2. (a) 226

Resolving 226 into its prime factors, we get

$$226 = 2 \times 113$$

2	226
113	113
	1

From the above, it is clear that all the prime factors are not grouped into pairs of identical factors.

So, 226 is not a perfect square.

(b) 100

Resolving 100 into its prime factors, we get

$$100 = \underline{2 \times 2} \times \underline{5 \times 5}$$

2	100
2	50
5	25
5	5
	1

From the above, it is clear that all the prime factors are grouped into pairs of identical factors, and no factor is leftover.

Hence, 100 is a perfect square.

(c) 121

Resolving 121 into its prime factors, we get

$$121 = 11 \times 11$$

11	121
11	11
	1

From the above, it is clear that all the prime factors are grouped into pairs of identical factors, and no factors is leftover.

Hence, 121 is a perfect square.

(d) 299

Resolving 299 into its prime factors, we get

$299 = 13 \times 23$

From the above, it is clear that all the prime factors are not grouped into pairs of identical factors.

So, 299 is not a perfect square.

13	299
23	23
	1

(e) 324

Resolving 324 into its prime factors, we get

$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

From the above, it is clear that all the prime factors are grouped into pairs of identical factors. And, no factors is leftover.

Hence, 324 is a perfect square.

2	324
2	162
3	81
3	27
3	9
3	3
	1

(f) 1024

Resolving 1024 into its prime factors, we get

$1024 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

From the above, it is clear that all the prime factors are grouped into pairs of identical factors. And, no factors is leftover.

Hence, 1024 is a perfect square.

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(g) 2027

Resolving 2027 into its prime factors, we get

$2027 = 2027 \times 1$

From the above, it is clear that all the prime factors are grouped into pairs of identical factors.

So, 2027 is not a perfect square.

2027	2027
	1

(h) 10404

Resolving 10404 in its prime factors, we get

$10404 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{17 \times 17}$

From the above, it is clear that all the prime factors are not grouped into pairs of identical factors. And, no factors is leftover.

Hence, 10404 is a perfect square.

2	10404
2	5202
3	2601
3	867
17	289
17	17
	1

3. We know that a number ending in 2, 3, 7 or 8 is never a perfect square.
So, (b) 233, (c) 188, (f) 697, and (g) 228 are not perfect squares.
$\because$ 2205 is not ending with 25.
So, (h) 2205 is also not a perfect square.

4. Since, a number ending with odd number of zeroes is not a perfect square.
So, (b) 2500000, (c) 9000, (f) 169000 and (h) 81000 are not perfect squares.

5. We know that, square root of odd number is odd and square root of even number is even.
So, square of even numbers = (b) 36, (d) 196, (e) 144 and (g) 100;
And square of odd numbers = (a) 169, (c) 225, (f) 625 and (h) 1225

6. From the property : $(n+1)^2 - n^2 = 2n+1$

(a) $98^2 - 97^2$
$= 2\times 97+1$
$= 194+1 = 195$

(b) $44^2 - 43^2$
$= 2\times 43+1$
$= 86+1 = 87$

(c) $80^2 - 79^2$
$= 2\times 79+1$
$= 158+1 = 159$

(d) $36^2 - 35^2$
$= 2\times 35+1$
$= 70+1 = 71$

(e) $105^2 - 104^2$
$= 2\times 104+1$
$= 208+1 = 209$

(f) $147^2 - 146^2$
$= 2\times 146+1$
$= 292+1 = 293$

(g) $238^2 - 237^2$
$= 2\times 237+1$
$= 474+1 = 475$

(h) $269^2 - 268^2$
$= 2\times 268+1$
$= 536+1 = 537$

7. (a) (4, 6, 8)
Three natural numbers a, b and c are called pythagorean triplet if
$$a^2 + b^2 = c^2$$
So, $$4^2 + 6^2 = 8^2$$
$$16 + 36 = 64$$
$$52 \neq 64$$
Hence, (4, 6, 8) are not pythagorean triplets.

(b) (6, 8, 10)
Three natural numbers a, b and c are called pythagorean triplets if
$$a^2 + b^2 = c^2$$
So, $$6^2 + 8^2 = 10^2$$
$$36 + 64 = 100$$
$$100 = 100$$
Hence, (6, 8, 10) are pythagorean triplets.

(c) (9, 81, 82)
Three natural numbers a, b and c are called pythagorean triplets if
$$a^2 + b^2 = c^2$$
So, $$9^2 + 81^2 = 82^2$$
$$81 + 6561 = 6724$$
$$6642 \neq 6724$$
Hence, (9, 81, 82) are not pythagorean triplets.

(d) (10, 24, 26)
Three natural numbers a, b and c are called pythagorean triplets if
$$a^2 + b^2 = c^2$$
So, $$10^2 + 24^2 = 26^2$$
$$100 + 576 = 676$$

$$676 = 676$$

Hence, (10, 24, 26) are pythagorean triplets.

(e) (15, 85, 87)

Three natural numbers a, b and c are called pythagorean triplets, if

$$a^2 + b^2 = c^2$$

So, $$15^2 + 85^2 = 87^2$$

$$225 + 7225 = 7569$$

$$7450 \neq 7569$$

Hence, (15, 85, 87) are not pythagorean triples.

(f) (26, 168, 170)

Three natural numbers, a, b and c are called pythagorean triplets, if

$$a^2 + b^2 = c^2$$

So, $$26^2 + 168^2 = 170^2$$

$$676 + 28224 = 28900$$

$$28900 = 28900$$

Hence, (26, 168, 170) are pythagorean triplets.

(g) (30, 224, 226)

Three natural numbers a, b and c are called pythagorean triplets, if

$$a^2 + b^2 = c^2$$

So, $$30^2 + 224^2 = 226^2$$

$$900 + 50176 = 51076$$

$$51076 = 51076$$

Hence, (30, 224, 226) are phythagorean triplets.

(h) (42, 440, 442)

Three natural numbers. a, b and c are called phythagorean triplets, if

$$a^2 + b^2 = c^2$$

So, $$42^2 + 440^2 = 442^2$$

$$1764 + 193600 = 195364$$

$$195364 = 195364$$

Hence, (42, 440, 442) are phythagorean triplets.

8. (a) $1 + 3 + 5 + 7 + \mathbf{9} = 5^2$

(b) $1 + 3 + 5 + 7 + 9 + \mathbf{11} = 6^2$

(c) $1 + 3 + 5 + 7 + 9 + 11 + \mathbf{13} = 7^2$

(d) $1 + 3 + 5 + 7 + 9 + 11 + 13 + \mathbf{15} = 8^2$

9. (a) $\because 121 = 11^2$

So, Sum of first 11 odd numbers.

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 11^2$

(b) $\because 169 = 13^2$

So, Sum of first 13 odd numbers.

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$

$= 13^2$

(c) $289 = (17)^2$

Sum of first 17 odd numbers.

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 = 17^2$

10. (a) $11^2 = 121$

$101^2 = 10201$

$1001^2 = 1002001$

$10001^2 = 1\mathbf{0002000}1$

$100001^2 = 1\mathbf{000020000}1$

$1000001^2 = 1\mathbf{00000200000}1$

$10000001^2 = 1\mathbf{0000002000000}1$

(b) $11^2 = 1\ 2\ 1$

$101^2 = 10201$

$10101^2 = 102030201$

$1010101^2 = \mathbf{1020304030201}$

$\mathbf{101010101} = 10203040504030201$

11. (a) $\frac{55555^2}{123454321}$

$= \frac{(5 \times 11111)^2}{123454321}$

$= \frac{5 \times 5 \times 11111 \times 11111}{123454321}$

$= \frac{25 \times 123454321}{123454321} = 25$

(b) $\frac{7777777^2}{1234567654321}$

$= \frac{(7 \times 1111111)^2}{1234567654321}$

$= \frac{7 \times 7 \times 1111111 \times 1111111}{1234567654321}$

$= \frac{49 \times 1234567654321}{1234567654322} = 49$

Exercise 6.2

1. (a) If $8 \times 8 = 64$, then $\sqrt{64}$ is **8**.

(b) If 11×11 is 121, then $\sqrt{121}$ is **11**.

(c) If 25×25 is 625, then $\sqrt{625}$ is **25**.

(d) If 15^2 is 225, then $\sqrt{225}$ is **15**.

(e) If $(2 \times 9)(2 \times 9)$ is 324, then $\sqrt{324}$ is $\mathbf{(2 \times 9) = 18}$.

(f) If $(2 \times 3 \times 5)(2 \times 3 \times 5)$ is 900, then $\sqrt{900}$ is $\mathbf{(2 \times 3 \times 5) = 30}$.

(g) If $(2 \times 3 \times 7)(2 \times 3 \times 7)$ is 1764, then $\sqrt{1764}$ is $\mathbf{(2 \times 3 \times 7) = 42}$.

(h) If $(7 \times 11 \times 13)^2 = 1002001$, then $\sqrt{1002001}$ is $\mathbf{(7 \times 11 \times 13) = 1001}$.

2. (a) 169

$169 = \underline{13 \times 13}$

Hence, $\sqrt{169} = \sqrt{\underline{13 \times 13}} = 13$

13	169
13	13
	1

(b) 625

$625 = \underline{5 \times 5} \times \underline{5 \times 5}$

Hence, $\sqrt{625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5}}$

$= 5 \times 5 = 25$

5	625
5	125
5	25
5	5
	1

(c) 900

$900 = 2\times 2\times 3\times 3\times 5\times 5$

Hence, $\sqrt{900} = \sqrt{\underline{2\times 2}\times \underline{3\times 3}\times \underline{5\times 5}}$

$= 2\times 3\times 5$

$= 30$

2	900
2	450
3	225
3	75
5	25
5	5
	1

(d) 1600

$1600 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{5\times 5}$

Hence, $\sqrt{1600} = 2\times 2\times 2\times 5$

$= 40$

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

(e) 2916

$2916 = \underline{2\times 2}\times \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}$

Hence, $\sqrt{2916} = \sqrt{\underline{2\times 2}\times \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}}$

$= 2\times 3\times 3\times 3$

$= 54$

2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(f) 7744

$7744 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{11\times 11}$

Hence, $\sqrt{7744} = \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{11\times 11}}$

$= 2\times 2\times 2\times 11$

$= 88$

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

(g) $\sqrt{8281}$

$8281 = \underline{7\times 7}\times \underline{13\times 13}$

Hence, $\sqrt{8281} = \sqrt{\underline{7\times 7}\times \underline{13\times 13}}$

$= 7\times 13 = 91$

7	8281
7	1183
13	169
13	13
	1

(h) 8464

$8464 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{23\times 23}$

Hence, $\sqrt{8464} = \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{23\times 23}}$

$= 2\times 2\times 23$

$= 92$

2	8464
2	4232
2	2116
2	1058
23	529
23	23
	1

(i) 9216

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144

2	72
2	36
2	18
3	9
3	3
	1

$9216 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{3\times 3}$

Hence, $\sqrt{9216} = \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{3\times 3}}$

$= 2\times 2\times 2\times 2\times 2\times 3 = 96$

(j) 10000

$10000 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{5\times 5}\times \underline{5\times 5}$

Hence, $\sqrt{10000} = \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{5\times 5}\times \underline{5\times 5}}$

$= 2\times 2\times 5\times 5$

$= 100$

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

(k) 11025

$11025 = \underline{3\times 3}\times \underline{5\times 5}\times \underline{7\times 7}$

Hence, $\sqrt{11025} = \sqrt{\underline{3\times 3}\times \underline{5\times 5}\times \underline{7\times 7}}$

$= 3\times 5\times 7$

$= 105$

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

(l) 12544

$12544 = \underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{7\times 7}$

Hence,

$\sqrt{12544} = \sqrt{\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{7\times 7}}$

$= 2\times 2\times 2\times 2\times 7$

$= 112$

2	12544
2	6272
2	3136
2	1568
2	784
2	392

2	196
2	98
7	49
7	7
	1

(m) 27225

$27225 = \underline{3\times 3}\times\underline{5\times 5}\times\underline{11\times 11}$

Hence, $\sqrt{27225} = \sqrt{\underline{3\times 3}\times\underline{5\times 5}\times\underline{11\times 11}}$

$= 3\times 5\times 11$

$= 165$

3	27225
3	9075
5	3025
5	605
11	121
11	11
	1

(n) 57600

2	57600
2	28800
2	14400
2	7200
2	3600
2	1800
2	900

2	450
3	225
3	75
5	25
5	5
	1

$57600 = \underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{3\times 3}\times\underline{5\times 5}$

Hence, $\sqrt{57600} = \sqrt{\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times\underline{3\times 3}\times\underline{5\times 5}}$

$= 2\times 2\times 2\times 2\times 3\times 5$

$= 240$

(o) 67600

$67600 = \underline{2\times 2}\times\underline{2\times 2}\times\underline{5\times 5}\times\underline{13\times 13}$

Hence, $\sqrt{67600}$

$= \sqrt{\underline{2\times 2}\times\underline{2\times 2}\times\underline{5\times 5}\times\underline{13\times 13}}$

$= 2\times 2\times 5\times 13$

$= 260$

2	67600
2	33800
2	16900
2	8450
5	4225
5	845
13	169
13	13
	1

(p) 99225

$99225 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}$

Hence, $\sqrt{99225}$

$= \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$

$= 3 \times 3 \times 5 \times 7$

$= 315$

3	99225
3	33075
3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

3. Let no. of plants in *a* row $= x$

and no. of plants in a columns $= x$

So, total number of plant in the garden

$= x \times x = x^2 = 2500$

$x = \sqrt{2500}$

$x = \sqrt{\underline{2 \times 2} \times \underline{5 \times 5} \times \underline{5 \times 5}}$

$= 2 \times 5 \times 5 = 50$

So, No. of plants in each row and column $= 50$.

And, no. of plants in each column = 50.

4. Base of triangle $= 3$ cm;

height of triangle $= 4$ cm;

hypotenuse $= ?$

By pythagoras theorem, we get

$\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$

$= 3^2 + 4^2$

$\text{hypotenuse}^2 = 9 + 16$

$\text{hypotenuse} = \sqrt{9+16} = \sqrt{25} = 5$ cm.

Hence, the length of the hypotenuse of triangle is 5 cm.

5. Finding all the prime factors of 338, we get

$338 = 2 \times \overline{13 \times 13}$

From the above, it is clear that 2 is leftover without pair.

So, 338 must be multiplied by 2 to make it a perfect square.

Verification : $338 \times 2 = 676$

(which is perfect square of 26.)

2	338
13	169
13	13
	1

6. Finding all the prime factors of 1575, we get

$1575 = \underline{3 \times 3} \times \underline{5 \times 5} \times 7$

From the above, it is clear that 7 is leftover without pair.

So, 1575 is to be divided by 7 to make it a perfect square.

Verification : $1575 \div 7 = 225$ is the perfect square of 15.

3	1575
3	525
5	175
5	35
7	7
	1

7. The least number which will be divisible by 10, 12 and 8 is the L.C.M of 10, 12 and 8.

2	10, 12, 8
2	5, 6, 4
2	5, 3, 2
3	5, 3, 1
5	5, 1, 1
	1, 1, 1

2	120
2	60
2	30
3	15
5	5
	1

LCM of 10, 12 and 8 is $(2\times 2\times 2\times 3\times 5)$ i.e., 120

Now, $120 = \underline{2\times 2}\times 2\times 3\times 5$

To make 120 a perfect square, it must be multiplied by $(2\times 3\times 5)$ *i.e.*, 30.

Hence, the required least number is (120×30), *i.e.*, 3600.

8. Size of a tile $= 3\times 3$ inch^2

Area of the roof $= 9\times 9$ feet^2

So, the no. of required tiles $= \dfrac{\text{Area of roof}}{\text{Size of one tile}}$

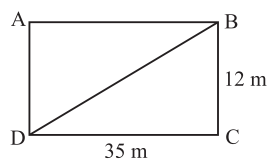
$$= \frac{9\times 9 \text{ feet}^2}{3\times 3 \text{ inch}^2} = \frac{9\times 9\times 12\times 12 \text{ inch}^2}{3\times 3 \text{ inch}^2}$$

$= 9\times 144 = 1296$ tiles.

9. In ΔBCD

$BC = 12$ m; $CD = 35$ m

Diagonal $BD = ?$



By Pythagoras theorem, we have

$$BD^2 = BC^2 + CD^2$$
$$BD^2 = 12^2 + 35^2$$
$$BD^2 = 144 + 1225$$
$$BD^2 = 1369$$
$$BD = \sqrt{1369}$$
$$BD = 37 \text{ m}$$

So, the length of the diagonal of rectangle is 37 m.

10. Side of the square field $= 125$ m

$\therefore$ Area of the square field $=$ side $\times$ side $= 125$ m $\times$ 125 m $= 15625$ m^2

Rate of leveling = ₹ 17 per m^2

So, the cost of leveling the entire ground = ₹ 15625 × 17 = ₹ 265625

11. Let the no. of students in the class $= x$

So, each student contributed rupees = ₹ x

$\because$ total contributed amount = ₹ 1024

$\therefore$ $x\times x = 1024$

$$x^2 = 1024$$
$$x = \sqrt{1024}$$

$$x = \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}}$$

$$x = 2\times 2\times 2\times 2\times 2$$

$$x = 32$$

So, there are 32 students in the class.

Exercise 6.3

1. (a) 529

	23
2	$\overline{5}\ \overline{29}$
+ 2	–4
43	129
+ 3	–129
	×

$\therefore \sqrt{529} = 23$

(b) 1444

	38
3	$\overline{14}\ \overline{44}$
+3	–9
68	544
+8	–544
	×

$\therefore \sqrt{1444} = 38$

(c) 1156

	34
3	$\overline{11}\ \overline{56}$
+ 3	–9
64	256
+ 4	–256
	×

$\therefore \sqrt{1156} = 34$

(d) 12544

	112
1	$\overline{1}\ \overline{25}\ \overline{44}$
+1	–1
21	25
+1	–21
222	444
+2	– 444
	×

$\therefore \sqrt{12544} = 112$

(e) 974169

	987
9	$\overline{97}\ \overline{41}\ \overline{69}$
+ 9	– 81
188	1641
+ 8	1504
1967	13769
7	13769
	×

Hence, $\sqrt{974169} = 987$

(f) 12321

	111
1	$\overline{1}\ \overline{23}\ \overline{21}$
+ 1	–1
21	23
+ 1	– 21
221	221
1	– 221
	×

Hence, $\sqrt{12321} = 111$

(g) 352836

	594
5	$\overline{35}\ \overline{28}\ \overline{36}$
+ 5	− 25
109	1028
+ 9	− 981
1184	4736
4	− 4736
	×

Hence, $\sqrt{352836} = 594$

(h) 363609

	603
6	$\overline{36}\ \overline{36}\ \overline{09}$
+ 6	−36
120	36
+ 0	− 00
1203	3609
	− 3609
	×

Hence, $\sqrt{363609} = 603$

(i) 17956

	134
1	$\overline{1}\ \overline{79}\ \overline{56}$
+1	−1
23	79
+3	− 69
264	1056
4	−1056
	×

Hence, $\sqrt{17956} = 134$

(j) 92416

	304
3	$\overline{9}\ \overline{24}\ \overline{16}$
+ 3	−9
60	24
+ 0	00
604	2416
4	−2416
	×

Hence, $\sqrt{92416} = 304$

2. Greatest no. of four digits $= 9999$

Our purpose is to find the least no. which when subtracted from 9999, gives a perfect square.

Now, on finding the square root of 9999 by long division method, we have

	99
9	$\overline{99}\ \overline{99}$
+ 9	− 81
189	1899
9	− 1701
	198

$$99^2 + 198 = 9999$$

So, the least number to be subtracted is 198.

Hence, the required number $= 9999 - 198 = 9801$

Also, $\sqrt{9801} = 99.$

3. Finding square root of 650 by long division method and on comparing, we have

	25
2	$\overline{6}\ \overline{50}$
+ 2	− 4
45	250
5	−225
	25

$$25^2 < 650 < 26^2$$

The required no. to be added $= (25 + 26) - 25$

Hence, $= 51 - 25 = 26$

4. Finding square root of 17455 by long division method, we have
Here, 31 is the remainder.
So, the required no. to be subtracted is 31 to make 17455 a perfect square.
$\therefore \quad 17455 - 31 = 17424 = 132^2$

	132
1	$\overline{1}\ \overline{74}\ \overline{55}$
+1	–1
23	74
+3	– 69
262	555
	–524
	31

5. Least no. of five digits $= 10000$
Find square root of 10000 by long division method, we have
Now, no. remainder is left. Hence, 10000 is a perfect square
$\therefore \quad \sqrt{10000} = 100$

	100
1	$\overline{1}\ \overline{00}\ \overline{00}$
+1	–1
20	00
	–00
200	00
	– 00
	×

6. Find square root of 60509 by long division method and on comparing, we have $\quad 245^2 < 60509 < 246^2$

	245
2	$6\ \overline{05}\ 0\ \overline{9}$
+2	–4
44	205
+4	–176
485	2909
5	–2425
	484

Hence, the required no. to be added $= (245 + 246) - 484 = 491 - 484 = 7$

7. (a) $\because$ Number of digits in number 225 is 3.
$\therefore$ Number of digits in square root of 225 is 2.
(b) $\because$ Number of digits in number 625 is 3.
$\therefore$ Number of digits in square root of 625 is 2.
(c) $\because$ Number of digits in number 10,000 is 5.
$\therefore$ Number of digits in square root of 10,000 is 3.
(d) $\because$ Number of digits in number 63,001 is 5.
$\therefore$ Number of digits in square root of 63,001 is 3.

8. The greatest no. of 7-digit $= 9999999$

Our purpose is to find the least no. which when subtracted from 9999999, gives a perfect square.

Now, on finding the square root of 9999999 by long division method, we have

$3162^2 + 1755 = 9999999$

So, the least no. to be subtracted is 1755

$\therefore \quad 9999999 - 1755 = 9998244$

Also, $\quad \sqrt{9998244} = 3162$

	3162
3	$\overline{9}\,\overline{99}\,\overline{99}\,\overline{99}$
+3	–9
61	99
+1	– 61
626	3899
+6	–3756
6322	14399
	–12644
	1755

9. Least no. of eight-digits $= 10000000$

Finding square root of 10000000 by long division method and on comparing,

we have

$$3162^2 < 10000000 < 3163^2$$

So, the required no. to be added

$= (3162 + 3163) - 1756$

$= 6325 - 1756$

$= 4569$

$\therefore$ The required perfect square

$= 10000000 + 4569$

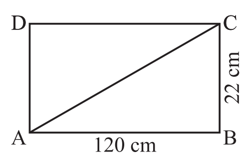
$= 10004569$

Therefore, $\sqrt{10004569} = 3163$

	3162
3	$\overline{10}\,\overline{00}\,\overline{00}\,\overline{00}$
+3	–9
61	100
+1	–61
626	3900
+6	– 3756
6322	14400
2	– 12644
	1756

10. In ΔABC, $AB = 120$ cm; $BC = 22$ cm; diagonal $AC = ?$

By Pythagoras theorem, we have



$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 120^2 + 22^2$$
$$AC^2 = 14400 + 484$$
$$AC^2 = 14884$$
$$AC = \sqrt{14884} = 122\,\text{cm}$$

	122
1	$\overline{1}\,\overline{48}\,\overline{84}$
+ 1	–1
22	48
+ 2	– 44
242	484
2	– 484
	×

So, the length of the diagonal of rectangle is 122 cm.

11. Area of square field $= 119025 \text{ m}^2$

Let its side be x

So, $x^2 = 119025 \text{ m}^2$

$x = \sqrt{119025}$ m

$= 345$ m

Now, perimeter of square $= 4 \times x$

$= 4 \times 345$ m

$= 1380$ m

So, distance covered by a man in 2 rounds of its boundary

$= 2 \times 1380 \text{ m} = 2760$ m.

	345
3	$\overline{11}\,\overline{90}\,\overline{25}$
+ 3	–9
64	2 90
+ 4	–2 56
685	3425
+ 5	–3425
	×

12. Area of square field $= 164025 \text{ m}^2$

Let, side of square field be $= x$ m

So, area $= 164025 \text{ m}^2$

or $x^2 = 164025 \text{ m}^2$

$x = \sqrt{164025}$ m

or $x = 405$ m

Now, perimeter of square field $= 4 \times x$

$= 4 \times 405$ m

$= 1620$ m

So, the cost of fencing the field $=$ ₹ 15×1620

$=$ ₹ 24300.

	405
4	$\overline{16}\,\overline{40}\,\overline{25}$
+4	–16
80	40
+0	– 00
805	4025
+5	– 4025
	×

Exercise 6.4

1. (a) $\sqrt{\dfrac{4096}{14641}} = \dfrac{\sqrt{4096}}{\sqrt{14641}}$

	64
6	$\overline{40}\,\overline{96}$
+6	–36
124	496
4	–496
	×

	121
1	$\overline{1}\,\overline{46}\,\overline{41}$
+1	–1
22	46
+2	–44
241	241
1	– 241
	×

So, $\sqrt{\dfrac{4096}{14641}} = \dfrac{64}{121}$

(b) $\sqrt{\dfrac{50625}{1444}} = \dfrac{\sqrt{50625}}{\sqrt{1444}}$

	225
2	$\overline{5}\ \overline{06}\ \overline{25}$
+2	–4
42	106
+2	– 84
445	2225
5	– 2225
	×

	38
3	$\overline{14}\ \overline{44}$
+3	– 9
68	544
8	–544
	×

So, $\sqrt{\dfrac{50625}{1444}} = \dfrac{225}{38} = 5\dfrac{35}{38}$

(c) $\sqrt{80\dfrac{244}{729}} = \sqrt{\dfrac{58564}{729}} = \dfrac{\sqrt{58564}}{\sqrt{729}}$

	242
2	$\overline{5}\ \overline{85}\ \overline{64}$
+ 2	– 4
44	185
+ 4	–176
482	964
	– 964
	×

	27
2	$\overline{7}\ \overline{29}$
+ 2	– 4
47	329
	–329
	×

So, $\sqrt{80\dfrac{244}{729}} = \sqrt{\dfrac{58564}{729}} = \dfrac{242}{27} = 8\dfrac{26}{27}$

(d) $\sqrt{407\dfrac{37}{121}} = \sqrt{\dfrac{49284}{121}} = \dfrac{\sqrt{49284}}{\sqrt{121}}$

	222
2	$\overline{4}\ \overline{92}\ \overline{84}$
+2	–4
42	92
+2	– 84
442	884
	– 884
	×

	11
1	$\overline{1}\ \overline{21}$
+1	– 1
21	21
+1	– 21
	×

So, $\sqrt{407\dfrac{37}{121}} = \sqrt{\dfrac{49284}{121}} = \dfrac{222}{11} = 20\dfrac{2}{11}$

(e) $\sqrt{23\frac{394}{729}} = \sqrt{\frac{17161}{729}} = \frac{\sqrt{17161}}{\sqrt{729}}$

	131
1	$\overline{1}\ \overline{71}\ \overline{61}$
+ 1	–1
23	71
+ 3	69
261	261
	261
	×

	27
2	$\overline{7}\ \overline{29}$
+2	– 4
47	329
	329
	×

So, $\sqrt{23\frac{394}{729}} = \sqrt{\frac{17161}{729}} = \frac{131}{27} = 4\frac{23}{27}$

(f) $\sqrt{21\frac{2797}{3364}} = \sqrt{\frac{73441}{3364}} = \frac{\sqrt{73441}}{\sqrt{3364}}$

	271
2	$\overline{7}\ \overline{34}\ \overline{41}$
+ 2	– 4
47	334
+ 7	–329
541	541
+ 1	–541
	×

	58
5	$\overline{33}\ \overline{64}$
+ 5	–25
108	864
	864
	×

So, $\sqrt{21\frac{2797}{3364}} = \sqrt{\frac{73441}{3364}} = \frac{271}{58} = 4\frac{39}{58}$

2. (a) $\sqrt{\frac{405}{180}} = \sqrt{\frac{3\times 3\times 3\times 3\times 5}{2\times 2\times 2\times 2\times 5}} = \sqrt{\frac{3\times 3}{2\times 2}} = \frac{3}{2}$

So, the square root of $\frac{405}{180}$ is $\frac{3}{2}$.

(b) $\sqrt{\frac{625}{441}} = \sqrt{\frac{\underline{5\times 5}\times \underline{5\times 5}}{\underline{3\times 3}\times \underline{7\times 7}}} = \frac{5\times 5}{3\times 7} = \frac{25}{21}$

So, the square root of $\frac{625}{441}$ is $\frac{25}{21}$.

(c) $\sqrt{\frac{1587}{1728}} = \sqrt{\frac{3\times \underline{23\times 23}}{2\times 2\times 2\times 2\times 2\times 2\times 3\times 3\times 3}}$

$= \sqrt{\frac{23\times 23}{\underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{3\times 3}}}$

$= \frac{23}{2\times 2\times 2\times 3} = \frac{23}{24}$

So, the square root of $\frac{1587}{1728}$ is $\frac{23}{24}$.

(d) $\sqrt{338\times 72} = \sqrt{338}\times \sqrt{72}$

$= \sqrt{2\times \underline{13\times 13}}\times \sqrt{2\times 2\times 2\times 3\times 3}$

$= \sqrt{2}\times 13\times \sqrt{2}\times 2\times 3 = 13\times 6\times (\sqrt{2}\times \sqrt{2})$

$= 78\times 2 = 156$

So, the square root of (338×72) is 156.

(e) $\sqrt{980\times 1620} = \sqrt{980}\times \sqrt{1620}$

$= \sqrt{2\times \underline{7\times 7}\times 10}\times \sqrt{2\times \underline{3\times 3}\times \underline{3\times 3}\times 10}$

$= 7\times \sqrt{20}\times 3\times 3\times \sqrt{20}$

$= 63\times (\sqrt{20}\times \sqrt{20}) = 63\times 20 \Rightarrow 1260$

Hence, the square root of (980×1620) is 1260.

(f) $\sqrt{\frac{1183}{2023}} = \sqrt{\frac{7\times 13\times 13}{7\times 17\times 17}} = \sqrt{\frac{13\times 13}{17\times 17}} = \frac{13}{17}$

So, the square root of $\frac{1183}{2023}$ is $\frac{13}{17}$.

(g) $\sqrt{147\times 243} = \sqrt{147}\times \sqrt{243}$

$= \sqrt{3\times \underline{7\times 7}}\times \sqrt{\underline{3\times 3}\times \underline{3\times 3}\times 3}$

$= 7\times \sqrt{3}\times 3\times 3\times \sqrt{3}$

$= (7\times 3\times 3)\times (\sqrt{3}\times \sqrt{3})$

$= 63\times 3 \Rightarrow 189$

So, the square root of (147×243) is 189.

3. Area of square sheet $= 3\frac{942}{2209}\ m^2$

Let side of square sheet $= x$

So, $x\times x = 3\frac{942}{2209}$

$x^2 = 3\frac{942}{2209}$

$x = \sqrt{3\frac{942}{2209}} = \sqrt{\frac{7569}{2209}}$

$= \frac{\sqrt{7569}}{\sqrt{2209}} = \frac{87}{47} = 1\frac{40}{47}$

	87
8	$\overline{75}\ \overline{69}$
+8	–64
167	1169
7	–1169
	×

	47
4	$\overline{22}\ \overline{09}$
+4	–16
87	609
7	– 609
	×

So, the side of square sheet of plywood is $1\frac{40}{47}$ m.

4. Area of square field $= 332\frac{61}{169}$ m^2

Let the side of square $= x$ m

$$\therefore \quad x \times x = 332\frac{61}{169}$$

$$x^2 = 332\frac{61}{169}$$

$$x = \sqrt{332\frac{61}{169}}$$

$$x = \sqrt{\frac{56169}{169}} = \frac{\sqrt{56169}}{\sqrt{169}} = \frac{237}{13} = 18\frac{3}{13}$$

	237
2	$\overline{5}\ \overline{61}\ \overline{69}$
+ 2	–4
43	161
+ 3	–129
467	3269
7	– 3269
	×

	13
1	$\overline{1}\ \overline{69}$
+ 1	–1
23	69
3	– 69
	×

So, the side of square field is $18\frac{3}{13}$ m.

5. (a) $\sqrt{1.4641}$ (b) $\sqrt{6.7081}$

	1.21
1	$\overline{1}.\overline{46}\ \overline{41}$
+ 1	–1
22	46
+ 2	– 44
241	241
1	– 241
	×

	2.59
2	$\overline{6}.\overline{70}\ \overline{81}$
+ 2	–4
45	270
+ 5	–225
509	4581
9	– 4581
	×

$\therefore \sqrt{1.4641} = 1.21$

$\therefore \sqrt{6.7081} = 2.59$

(c) $\sqrt{1011.24}$

	31.8
3	$\overline{10}\ \overline{11}.\overline{24}$
+ 3	− 9
61	111
+ 1	− 61
628	5024
8	− 5024
	×

$\therefore \sqrt{1011.24} = 31.8$

(d) $\sqrt{0.053361}$

	0.231
0	$\overline{0}.\overline{05}\ \overline{33}\ \overline{61}$
+ 0	− 0
2	5
+ 2	− 4
43	133
+ 3	−129
461	461
1	− 461
	×

$\therefore \sqrt{0.053361} = 0.231$

(e) $\sqrt{477.4225}$

	21.85
2	$\overline{4}\ \overline{77}.\overline{42}\ \overline{25}$
+ 2	− 4
41	77
+ 1	−41
428	3642
+ 8	− 3424
4365	21825
5	− 21825
	×

$\therefore \sqrt{477.4225} = 21.85$

(f) $\sqrt{0.00008649}$

	0.0093
0	$\overline{0}.\overline{00}\ \overline{00}\ \overline{86}\ \overline{49}$
+ 0	− 0
0	00 00
0	− 00 00
9	86
+ 9	− 81
183	549
3	−549
	×

$\therefore \sqrt{0.00008649} = 0.0093$

(g) $\sqrt{150.0625}$

	12.25
1	$\overline{1}\ \overline{50}.\overline{06}\ \overline{25}$
+ 1	−1
22	50
+ 2	− 44
242	606
+ 2	− 484
2445	12225

(h) $\sqrt{9998.0001}$

	99.99
9	$\overline{99}\ \overline{98}.\overline{00}\ \overline{01}$
+ 9	−81
189	1898
+ 9	−1701
1989	19700
+ 9	−17901
19989	179901

5	−12225
	×

$\therefore \sqrt{150.0625} = 12.25$

9	−179901
	×

$\therefore \sqrt{9998.0001} = 99.99$

(i) $\sqrt{493.7284}$

	22.22
2	$\overline{4}\ \overline{93}.\overline{72}\ \overline{84}$
+ 2	− 4
42	93
+ 2	−84
442	972
+ 2	−884
4442	8884
2	−8884
	×

$\therefore \sqrt{493.7284} = 22.22$

(j) $\sqrt{225.6004}$

	15.02
1	$\overline{2}\ \overline{25}.\overline{60}\ \overline{04}$
+ 1	−1
25	125
+ 5	−125
300	60
+ 0	− 00
3002	6004
2	− 6004
	×

$\therefore \sqrt{225.6004} = 15.02$

(k) $\sqrt{0.00015129}$

	0.0123
0	$\overline{0}.\overline{00}\ \overline{01}\ \overline{51}\ \overline{29}$
	0
0	00
	00
1	01
+ 1	−1
22	51
+ 2	− 44
243	729
3	−729
	×

$\therefore \sqrt{0.00015129} = 0.0123$

(l) $\sqrt{227.798649}$

	15.093
1	$\overline{2}\ \overline{27}.\overline{79}\ \overline{86}\ \overline{49}$
+ 1	−1
25	127
+ 5	−125
300	279
+ 0	−000
3009	27986
+ 9	−27081
30183	90549
3	90549
	×

$\therefore \sqrt{227.798649} = 15.093$

6. Area of a square field = 144.288144 m^2

Let the length of side $= x$ m

So, area of square field $= x \times x$

$\therefore \quad x \times x = 144.288144$

$x^2 = 144.288144$

$\Rightarrow \quad x = \sqrt{144.288144}$

$= 12.012$ m

Hence, the length of each side of the square field is 12.012 m.

	12.012
1	$\overline{1}\ \overline{44}.\overline{28}\ \overline{81}\ \overline{44}$
+1	–1
22	44
+2	– 44
240	28
0	– 00
2401	2881
+1	–2401
24022	48044
2	– 48044
	×

7. Let fraction $= x$

So, $x \times x = 0.00001156$

$x^2 = 0.00001156 \quad \Rightarrow \quad x = \sqrt{0.00001156}$

	0.0034
0	$\overline{0}.\overline{00}\ \overline{00}\ \overline{11}\ \overline{56}$
+ 0	0
0	00
+ 0	– 00
0	00
+ 0	– 00
3	11
+3	– 9
64	256
4	– 256
	×

$x = 0.0034$

$x = 0.0034 \times \frac{10000}{10000} = \frac{34}{10000} = \frac{17}{5000}$

Hence, the required fraction is $\frac{17}{5000}$.

8. (a) $\sqrt{0.09102289}$

$\therefore \quad \sqrt{910.2289} = 30.17$

(a) $\sqrt{0.09102289}$

We have, $\sqrt{910.2289} = 30.17$

So, $\sqrt{0.09102289} = \sqrt{\frac{910.2289}{10.000}} = \frac{30.17}{100}$

$= 0.3017$

	30.17
3	$\overline{9}\ \overline{10}.\overline{22}\ \overline{89}$
+ 3	– 9
60	10
+ 0	– 00
601	1022
+ 1	601
6027	42189
7	42189
	×

(b) $\sqrt{9.102289}$

We have, $\sqrt{910.2289} = 30.17$

So, $\sqrt{9.102289} = \sqrt{\dfrac{910.2289}{100}} = \dfrac{30.17}{10} = 3.017$

(c) $\sqrt{91022.89}$

We have, $\sqrt{910.2289} = 30.17$

So, $\sqrt{91022.89} = \sqrt{910.2289 \times 100} = 30.17 \times 10 = 301.7$

Multiple Choice Questions

1. (b) **2.** (b) **3.** (b) **4.** (d) **5.** (b)

Brain Teaser

1. (i) 27556

2	27556
2	13778
83	6889
83	83
	1

$\therefore\ 27556 = 2 \times 2 \times 83 \times 83$

$= 2 \times 83 = 166$

So, $\sqrt{27556} = \sqrt{\underline{2 \times 2} \times \underline{83 \times 83}}$

$= 2 \times 83 = 166$

Hence, the square root of 27556 is 166.

(ii) 30625

5	30625
5	6125
5	1225
5	245
7	49
7	7
	1

$\therefore\ 30625 = 5 \times 5 \times 5 \times 5 \times 7 \times 7$

So, $\sqrt{30625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5} \times \underline{7 \times 7}}$

$= 5 \times 5 \times 7$

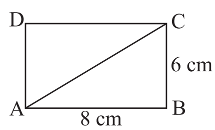
$= 175$

Hence, the square root of 305625 is 175.

2. Let, and $AB = 8$ cm; $BC = 6$ cm;

Diagonal $AC = ?$

By Pythagoras theorem, we have



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $AC^2 = 8^2 + 6^2$

$\Rightarrow$ $AC^2 = 64 + 36$

$\Rightarrow$ $AC^2 = 100$

$\Rightarrow$ $AC = \sqrt{100}$ cm

$\Rightarrow$ $AC = 10$ cm

So, the diagonal of the rectangle is 10 cm.

3. $\dfrac{\sqrt{0.2304} - \sqrt{0.1764}}{\sqrt{0.2304} + \sqrt{0.1764}}$

	0.48
0	$\overline{0}.\overline{23}\ \overline{04}$
+ 0	0
4	23

	0.42
0	$\overline{0}.\overline{17}\ \overline{64}$
+ 0	0
4	17

+ 4	–16
88	704
8	–704
	×

+ 4	–16
82	164
2	–164
	×

So, $\dfrac{\sqrt{0.2304}-\sqrt{0.1764}}{\sqrt{0.2304}+\sqrt{0.1764}}=\dfrac{0.48-0.42}{0.48+0.42}=\dfrac{0.06}{0.90}=\dfrac{6}{90}=\dfrac{1}{15}$

7 Cube and Cube Roots

Exercise 7.1

1. (a) $(75)^3 = 75\times75\times75 = 421875$

(b) $(99)^3 = 99\times 99\times 99 = 970299$

(c) $(104)^3 = 104\times 104\times 104 = 1124864$

(d) $(231)^3 = 231\times 231\times 231 = 12326391$

2. We know that cubes of odd numbers are odd.

So, (a) 6,859, (b) 4,913, (d) 2,197 and (f) 35,937 are cubes of odd natural numbers.

3. (a) 1,728

Writing 1728 as product of its prime factors, we get

2	1728
2	864
2	432
2	216
2	108

2	54
3	27
3	9
3	3
	1

$1728 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$

Grouping them into groups of three, we can see that no number is left ungrouped. So, 1728 is a perfect cube.

(b) 3,840

Writing 3840 as product of its prime factors, we get

2	3840
2	1920
2	960
2	480
2	240
2	120

2	60
2	30
3	15
5	5
	1

$3840 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times2\times2\times3\times5$

Grouping them into groups of three, we can see that 2, 3 and 5 are left ungrouped.

So, 3840 is not a perfect cube.

(c) 12,167

Writing 12167 as product of its prime factors,

we get $12167 = \underline{23\times23\times23}$

Grouping them into groups of three,

we can see that no number is left ungrouped.

So, 12,167 is a perfect cube.

23	12167
23	529
23	23
	1

(d) 11,109

Writing 11109 as product of its prime factors,

we get $11109 = 3\times7\times23\times23$

Grouping them into groups of three, we can see that all the prime factors are not in the group of three.

So, 11109 is not a perfect cube.

3	11109
7	3703
23	529
23	23
	1

(e) 85,184

Writing 85,184 as product of its prime factors, we get

2	85184
2	42592
2	21296
2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$85184 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{11\times11\times11}$

Grouping them into groups of three, we can see that no number is left ungrouped

So, 85184 is a perfect cube.

(f) 20,48,383

Writing 20,48,383 as product of its prime factors,

we get $2048383 = \underline{127\times127\times127}$

Grouping, them into group of three, we can see that no number is left ungrouped.

So, 20,48,383 is a perfect cube.

127	2048383
127	16129
127	127
	1

4. We know that cubes of even natural no. are even.

So, (a) 13,824, (c) 8,000, (d) 6,36,056 and (f) 32,768 are cubes of natural even numbers.

5. (a) 5,324

Writing 5324 as product of its prime factors,

we get $5324 = 2\times2\times\underline{11\times11\times11}$

Grouping them into groups of three, we see that 2 is

2	5324
2	2662
11	1331
11	121
11	11
	1

left ungrouped. So, to make it a perfect cube, complete the group by multiplying 5324 by 2.

So, the smallest number by which 5324 should be multiplied to make it a perfect cube is 2.

(b) 1,323

Writing 1323 as product of its prime factors,

we get $1323 = \underline{3 \times 3 \times 3} \times 7 \times 7$

Grouping them into groups of three, we see that 7 is left ungrouped. So, to make it a perfect cube, complete the group by multiplying 1323 by 7.

So, the smallest number by which 1323 should be multiplied to make it a perfect cube is 7.

3	1323
3	441
3	147
7	49
7	7
	1

(c) 27,783

Writing 27783 as product of its prime factors, we get

$27783 = \underline{3 \times 3 \times 3} \times 3 \times \underline{7 \times 7 \times 7}$

Grouping them into group of three, we see that 3 is left ungrouped. So, to make it a perfect cube, complete the group by multiplying 27783 by $(3 \times 3) = 9$.

So, the smallest number by which 27783 should be multiplied to make it a perfect cube is 9.

3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

(d) 12,348

Writing 12348, as product of its prime factors,

we get $12348 = 2 \times 2 \times 3 \times 3 \times \underline{7 \times 7 \times 7}$

Grouping them into groups of three, we see that 2 and 3 are left ungrouped.

So, to make it a perfect cube, complete the group by multiplying 12348 by $2 \times 3 = 6$.

So, the smallest number by which 12348 should be multiplied to make it a perfect cube is 6.

2	12348
2	6174
3	3087
3	1029
7	343
7	49
7	7
	1

(e) 3,125

Writing 3125, as a product of its prime factors,

we get $3125 = 5 \times 5 \times \underline{5 \times 5 \times 5}$

Grouping them into group of three, we see that 5 is left ungrouped. So, to make it a perfect cube, complete the group by multiplying 3125 by 5.

So, the smallest number by which 3125 should be multiplied to make it a perfect cube is 5.

5	3125
5	625
5	125
5	25
5	5
	1

(f) 1,01,306

Writing 101306, as product of its prime factors, we get $101306 = 2 \times \underline{37 \times 37 \times 37}$

2	101306
37	50653
37	1369
37	37
	1

Grouping them into groups of three, we see that 2 is left ungrouped.

So, to make it a perfect cube, complete the group by multiplying 101306 by 2 × 2 = 4.

So, the smallest number by which 101306 should be multiplied to make it a perfect cube is 4.

6. Edge of cubical box $= 13$ cm

So, the volume of cubical box $= (\text{edge})^3$

$= (13 \text{ cm})^3$

$= 13 \text{ cm} \times 13 \text{ cm} \times 13 \text{ cm}$

$= 2197 \text{ cm}^3$.

7. Surface area of the cube $= 294 \text{ m}^2$

Let the edge of the cube $= x$ m

Since, the surface area of the cube $= 6x^2$

So, $6x^2 = 294 \text{ m}^2$

$x^2 = \frac{294}{6} \text{ m}^2$

$x^2 = 49 \text{ m}^2$

$x = \sqrt{49} = 7 \text{ m}$

Hence, the Volume of the cube $= (\text{edge})^3 = (7\text{m})^3$

$= 7\text{m} \times 7\text{m} \times 7\text{m}$

$= 343 \text{ m}^3$.

8. (a) 1,536

Writing 1536 as product of its prime factors, we get $1536 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Grouping them into groups of three, we see that 3 is left ungrouped and should be removed to make 1536 a perfect cube.

So, the smallest number by which 1536 should be divided to make it a perfect cube is 3.

(b) 9,826

Writing 9826 as a product of its prime factors, we get $9826 = 2 \times \underline{17 \times 17 \times 17}$

2	9826
17	4913
17	289
17	17
	1

Grouping them into groups of three, we see that 2 is left ungrouped and should be removed to make 9826 a perfect cube.

So, the smallest number by which 9826 should be divided to make it a perfect cube is 2.

(c) 3,97,535

Writing 397535 as product of its prime factors, we get $397535 = 5 \times \underline{43 \times 43 \times 43}$

5	397535
43	79507
43	1849
43	43
	1

Grouping them into groups of three, we see that 5 is left ungrouped and should be removed to make 397535 a perfect cube.

So, the smallest number by which 397535 should be divided to make it a perfect cube is 5.

(d) 3,26,592

Writing 326592 as product of its prime factors, we get. $326592 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times 7$

2	326592
2	163296
2	81648
2	40824
2	20412
2	10206
3	5103
3	1701
3	567
3	189
3	63
3	21
7	7
	1

Grouping them into groups of three, we see that 7 is left ungrouped and should be removed to make 326592 a perfect cube.

So, the smallest number by which 326592 should be divided to make it a perfect cube is 7.

(e) 3,31,776

Writing 331776 as product of its prime factors, we get.

2	331776
2	165888
2	82944
2	41472
2	20736
2	10368
2	5184
2	2592
2	1296

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$331776 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 3$

Grouping them into groups of three, we see that 3 is left ungrouped and should be removed to make 331776 a perfect cube.

So, the smallest number by which 3,31,776 should be divided to make it a perfect cube is 3.

(f) 8,788

Writing 8788 as product of its prime factors,

we get $8788 = 2 \times 2 \times \underline{13 \times 13 \times 13}$

2	8788
2	4394
13	2197
13	169
13	13
	1

Grouping them into groups of three, we see that 2 is left ungrouped and should be removed to make 8788 a perfect cube.

So, the smallest number by which 8788 should be divided to make it a perfect cube is $2 \times 2 = 4$.

9. 4 natural numbers which are multiple of 5 are 5, 10, 15 and 20.

Now, cube of 5 $= 5 \times 5 \times 5 = 125$

cube of 10 $= 10 \times 10 \times 10 = 1000$

cube of 15 $= 15 \times 15 \times 15 = 3375$

And, cube of 20 $= 20 \times 20 \times 20 = 8000$

Now, we see that

$$125 \div 125 = 1$$
$$1000 \div 125 = 8$$
$$3375 \div 125 = 27$$
$$8000 \div 125 = 64$$

So, it is verified that the cube of a natural number which is a multiple of 5 is a multiple of 125.

10. 7 natural no. which are in the form of $3n+1$ are 4, 7, 10, 13, 16, 19 and 22

Now,
$$4^3 = 4 \times 4 \times 4 = 64$$
$$7^3 = 7 \times 7 \times 7 = 343$$
$$10^3 = 10 \times 10 \times 10 = 1000$$
$$13^3 = 13 \times 13 \times 13 = 2197$$
$$16^3 = 16 \times 16 \times 16 = 4096$$
$$19^3 = 19 \times 19 \times 19 = 6859$$
$$22^3 = 22 \times 22 \times 22 = 10648$$

Now,

$64 = 3 \times 21 + 1$ Remainder

$343 = 3 \times 114 + 1$ Remainder

$1000 = 3 \times 333 + 1$ Remainder

$2197 = 3 \times 732 + 1$ Remainder

$4096 = 3 \times 1365 + 1$ Remainder

$6859 = 3 \times 2286 + 1$ Remainder

$10648 = 3 \times 3549 + 1$ Remainder

So, from above, it is verified that the cube of a natural number of the form $3n+1$ when divided by 3 leaves remainder 1.

11. 7 natural numbers which are in the form of $3n+2$ are are 5, 8, 11, 14, 17, 20 and 23.

Now,
$$5^3 = 5 \times 5 \times 5 = 125$$
$$8^3 = 8 \times 8 \times 8 = 512$$
$$11^3 = 11 \times 11 \times 11 = 1331$$
$$14^3 = 14 \times 14 \times 14 = 2744$$

$$17^3 = 17 \times 17 \times 17 = 4913$$
$$20^3 = 20 \times 20 \times 20 = 8000$$
$$23^3 = 23 \times 23 \times 23 = 12167$$

Now, $125125 = 3 \times 41 + 2$ Remainder

$512 = 3 \times 170 + 2$ Remainder

$13131 = 3 \times 443 + 2$ Remainder

$2744 = 3 \times 914 + 2$ Remainder

$4913 = 3 \times 1637 + 2$ Remainder

$8000 = 3 \times 2666 + 2$ Remainder

$12167 = 3 \times 4055 + 2$ Remainder

So, from above, it is verified that the cube of a natural number of the form $(3n + 2)$ when divided by 3 leaves remainder 2.

12.
$$1^3 = 1$$
$$1^3 + 2^3 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$$
$$\therefore \quad \underline{1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2}$$
$$\underline{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = (1+2+3+4+5+6)^2}$$
$$\underline{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = (1+2+3+4+5+6+7)^2}$$

Now, $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$
$$= (1+2+3+4+5+6+7+8+9+10)^2$$
$$= 55^2 = 3025$$

13. (a) 2

Let no. be x.

Cube of this no. $= x^3$

Now, if no. is multiplied by 2, we get.

New no. $= 2x$

Cube of this no. $= (2x)^3 = 2 \times 2 \times 2 \times x^3 = 8x^3$

Hence the cube of new no. will be 8 times of cube of original no.

(b) 5

Let no. be x.

Cube of this no. $= x^3$

Now, if no. is multiplied by 5, we get

New no. $= 5x$

Cube of this no. $= (5x)^3 = 5 \times 5 \times 5 \times x^3 = 125x^3$

Hence. the cube of new no. will be 125 times of cube of original no.

(c) 7

Let no. be x.

Cube of this no. $= x^3$

Now, if no. is multiplied by 7, we get

New no. $= 7x$

Cube of this no. $= (7x)^3 = 7 \times 7 \times 7 \times x^3 = 343x^3$

Hence, the cube of new no. will be 343 times of cube of original no.

(d) 6

Let no. be x.

Cube of this no. $= x^3$

Now, if no. is multiplied by 6, we get

New no. $= 6x$

Cube of this no. $= (6x)^3 = 6\times 6\times 6\times x^3 = 216x^3$

Hence the cube of new no. will be 216 times of cube of original no.

14. (a) Unit digit of 33 is 3.

And, cube of 3 $= 3\times 3\times 3 = 27.$

So, the unit digit of the cube of 33 is 7.

(b) Unit digit of 132 is 2.

And, cube of 2 $= 2\times 2\times 2 = 8.$

So, the unit digit of the cube of 132 is 8.

(c) Unit digit of 995 is 5.

And, cube of $5 = 5\times 5\times 5 = 125.$

So, the unit digit of the cube of 195 is 5.

(d) Unit digit of 654 is 4.

And, cube of 4 $= 4\times 4\times 4 = 64.$

So, the unit digit of the cube of 654 is 4.

(e) Unit digit of 1999 is 9.

And, cube of 9 $= 9\times 9\times 9 = 729.$

So, the unit digit of the cube of 1999 is 9.

(f) Unit digit of 2008 is 8.

And, cube of 8 $= 8\times 8\times 8 = 512.$

So, the unit digit of the cube of 2008 is 2.

15. (a) $\{(5^2+12^2)^{1/2}\}^3$

$= \{(25+144)^{\frac{1}{2}}\}^3$

$= \{(169)^{\frac{1}{2}}\}^3$

$= \{(13^2)^{\frac{1}{2}}\}^3$

$= \{13^{2\times\frac{1}{2}}\}^3$

$= \{13\}^3$

$= 13\times 13\times 13$

$= 2197$

(b) $(\sqrt{10^3-6^3})^3$

$= (\sqrt{10\times 10\times 10-6\times 6\times 6})^3$

$= (\sqrt{1000-216})^3$

$= (\sqrt{784})^3$

$= (\sqrt{28\times 28})^3$

$= (28)^3$

$= 28\times 28\times 28$

$= 21952$

(c) $\{(10^2+24^2)^{1/2}\}^3$

$= \{(10\times 10+24\times 24)^{\frac{1}{2}})^3$

$= \{(100+576)^{\frac{1}{2}}\}$

$= \{(676)^{\frac{1}{2}}\}^3$

(d) $\{(37^2-35^2)^{1/2}\}^3$

$= \{(37\times 37-35\times 35)^{\frac{1}{2}}\}^3$

$= \{(1369-1225)^{\frac{1}{2}}\}^3$

$= \{(144)^{\frac{1}{2}}\}^3$

$= \{(26^2)^{\frac{1}{2}}\}^3$ $= \{(12^2)^{\frac{1}{2}}\}^3$

$= \{26^{2\times\frac{1}{2}}\}^3$ $= \{12^{2\times\frac{1}{2}}\}^3$

$= \{26\}^3$ $= \{12\}^3$

$= 26\times 26\times 26 = 17576$ $= 12\times 12\times 12 = 1728$

Exercise 7.2

1. (a) $(-12)^3$

$= (-12)\times(-12)\times(-12)$

$= -1728$

(b) $(-23)^3$

$= (-23)\times(-23)\times(-23)$

$= -12167$

(c) $(-35)^3$

$= (-35)\times(-35)\times(-35)$

$= -42875$

(d) $(-42)^3$

$= (-42)\times(-42)\times(-42)$

$= -74088$

2. We know that, cubes of negative numbers are negative.

So, (a) –1,331, (b) –4,913 and (d) –1,40,608 are cube of negative integers.

3. (a) $\sqrt[3]{-6,859}$

$= \sqrt[3]{-1}\times\sqrt[3]{6859}$

$= -1\times\sqrt[3]{19\times 19\times 19}$

$= -1\times 19$

$= -19$

19	6859
19	361
19	19
	1

(b) $\sqrt[3]{-97,336}$

$= \sqrt[3]{-1}\times\sqrt[3]{97336}$

$= -1\times\sqrt[3]{\underline{2\times 2\times 2}\times\underline{23\times 23\times 23}}$

$= -1\times 2\times 23$

$= -46$

2	97336
2	48668
2	24334
23	12167
23	529
23	23
	1

(c) $\sqrt[3]{-21952}$

$= \sqrt[3]{-1}\times\sqrt[3]{21952}$

$= -1\times\sqrt[3]{\underline{2\times 2\times 2}\times\underline{2\times 2\times 2}\times\underline{7\times 7\times 7}}$

$= -1\times 2\times 2\times 7$

$= -28$

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

(d) $\sqrt[3]{-1771561}$

$= \sqrt[3]{-1}\times\sqrt[3]{1771561}$

$= -1\times\sqrt[3]{\underline{11\times 11\times 11}\times\underline{11\times 11\times 11}}$

$= -1\times 11\times 11$

$= -121$

11	1771561
11	161051
11	14641
11	1331
11	121
11	11
	1

(e) $\sqrt[3]{-8,57,375}$

$= \sqrt[3]{-1} \times \sqrt[3]{857375}$

$= -1 \times \sqrt[3]{\underline{5\times5\times5}\times\underline{19\times19\times19}}$

$= -1\times 5\times 19$

$= -95$

5	857375
5	171475
5	34295
19	6859
19	361
19	19
	1

4. (a) $(6.3)^3 = 6.3 \times 6.3 \times 6.3$

$= \dfrac{63\times63\times63}{10\times10\times10} = \dfrac{250047}{1000} = 250.047$

(b) $(-0.21)^3 = (-0.21)\times(-0.21)\times(-0.21)$

$= \dfrac{(-21)\times(-21)\times(-21)}{100\times100\times100} = \dfrac{-9261}{1000000} = -0.009261$

(c) $\left(3\dfrac{1}{4}\right)^3 = \left(\dfrac{13}{4}\right)^3 = \dfrac{13\times13\times13}{4\times4\times4} = \dfrac{2197}{64} = 34\dfrac{21}{64}$

(d) $\left(-5\dfrac{1}{6}\right)^3 = \left(\dfrac{-31}{6}\right)^3 = \dfrac{(-31)\times(-31)\times(-31)}{6\times6\times6} = \dfrac{-29791}{216} = -137\dfrac{199}{216}$

(e) $(0.08)^3 = 0.08 \times 0.08 \times 0.08 = \dfrac{8\times8\times8}{100\times100\times100}$

$= \dfrac{512}{1000000} = 0.000512$

(f) $(-7.02)^3 = (-7.02)\times(-7.02)\times(-7.02)$

$= \dfrac{(-702)}{100}\times\dfrac{(-702)}{100}\times\dfrac{(-702)}{100} = \dfrac{-345948408}{1000000}$

$= -345.948408$

5. (a) $\sqrt[3]{\dfrac{1728}{4913}} = \dfrac{\sqrt[3]{1728}}{\sqrt[3]{4913}}$

By prime factorization method

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

17	4913
17	289
17	17
	1

$= \dfrac{\sqrt[3]{\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}}}{\sqrt[3]{17\times17\times17}} = \dfrac{2\times2\times3}{17} = \dfrac{12}{17}$

(b) $\sqrt[3]{\dfrac{-512}{2{,}197}} = -1 \times \sqrt[3]{\dfrac{512}{2197}} = -\dfrac{\sqrt[3]{512}}{\sqrt[3]{2197}}$

By prime factorization method.

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

13	2197
13	169
13	13
	1

$$= \frac{-\sqrt{\underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times \underline{2\times 2\times 2}}}{\sqrt{13\times 13\times 13}} = \frac{-(2\times 2\times 2)}{13} = \frac{-8}{13}$$

(c) $\sqrt[3]{0.551368} = \sqrt[3]{\dfrac{551368}{1000000}} = \dfrac{\sqrt[3]{551368}}{\sqrt[3]{1000000}}$

By prime factorization method

2	551368
2	275684
2	137842
41	68921
41	1681
41	41
	1

2	1000000
2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125

5	625
5	125
5	25
5	5
	1

$$= \frac{\sqrt[3]{\underline{2\times 2\times 2}\times \underline{41\times 41\times 41}}}{\sqrt[3]{\underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times \underline{5\times 5\times 5}\times \underline{5\times 5\times 5}}}$$

$$= \frac{2\times 41}{2\times 2\times 5\times 5}$$

$$= \frac{82}{100} = 0.82$$

(d) $\sqrt[3]{-0.024389} = -\sqrt[3]{0.024389}$

$$= -\sqrt[3]{\frac{24389}{1000000}} = \frac{-\sqrt[3]{24389}}{\sqrt[3]{1000000}}$$

By prime factorization method

29	24389
29	841
29	29
	1

2	1000000
2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$= \frac{-\sqrt[3]{\underline{29 \times 29 \times 29}}}{\sqrt{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}}}$$

$$= \frac{-29}{2 \times 2 \times 5 \times 5} = \frac{-29}{100} = -0.29$$

(e) $\sqrt[3]{-0.008} = -\sqrt[3]{0.008} = \frac{-\sqrt[3]{8}}{\sqrt[3]{1000}}$

By prime factorization method.

2	8
2	4
2	2
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$= \frac{-\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}} = \frac{-2}{2 \times 5} = \frac{-2}{10} = -0.2$$

Exercise 7.3

1. (a) 8

Subtract 1, 7, 19… and so on successively from 8, till you get 0.

$8 - 1 = 7$

$7 - 7 = 0$

It ends in two steps. So, cube root of $8 = 2$

i.e., $\sqrt[3]{8} = 2$

(b) 512

Subtract 1, 7, 19… and so on successively from 512, till you get 0.

$$512-1=511$$
$$511-7=504$$
$$504-19=485$$
$$485-37=448$$
$$448-61=387$$
$$387-91=296$$
$$296-127=169$$
$$169-169=0$$

It ends in eight steps. So, cube root of $512=8$

i.e., $\sqrt[3]{512}=8$

(c) 343

Subtract 1, 7, 19… and so on successively from 343, till you get 0.

$$343-1=342$$
$$342-7=335$$
$$335-19=316$$
$$316-37=279$$
$$279-61=218$$
$$218-91=127$$
$$127-127=0$$

It ends in seven steps. So, cube root of $343=7$

i.e., $\sqrt[3]{343}=7$

(d) 27

Subtract 1, 7, 19… and so on successively from 27, till you get 0.

$$27-1=26$$
$$26-7=19$$
$$19-19=0$$

It ends in three steps. So, cube root of $27=3$

i.e., $\sqrt[3]{27}=3$

(e) 125

Subtract 1, 7, 19… and so on successively from 125, till you get 0.

$$125-1=124$$
$$124-7=117$$
$$117-19=98$$
$$98-37=61$$
$$61-61=0$$

It ends in five steps. So, cube root of $125=5$

i.e., $\sqrt[3]{125}=5$

(f) 729

Subtract 1, 7, 19… and so on. Successively from 729, till you get 0.

$$729-1=728$$
$$728-7=721$$
$$721-19=702$$

$$702 - 37 = 665$$
$$665 - 61 = 604$$
$$604 - 91 = 513$$
$$513 - 127 = 386$$
$$386 - 169 = 217$$
$$217 - 217 = 0$$

It ends in nine steps. So cube root of $729 = 9$

i.e., $\sqrt[3]{729} = 9$

2. (a) 70

Subtract 1, 7, 19… and so on.

$$70 - 1 = 69$$
$$69 - 7 = 62$$
$$62 - 19 = 43$$
$$43 - 37 = 6$$

Here, we see remainder is not getting 0.

So 70 is not a perfect cube.

We have to subtract 6 from 70 to make it a perfect cube.

Now, $70 - 6 = 64$

and $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

(b) 128

Subtract 1, 7, 19… and so on.

$$128 - 1 = 127$$
$$127 - 7 = 120$$
$$120 - 19 = 101$$
$$101 - 37 = 64$$
$$64 - 61 = 3$$

Here, we see, remainder is not getting 0.

So, 128 is not a perfect cube.

We have to subtract 3 from 128 to make it a perfect cube.

Now, $128 - 3 = 125$

and $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

(c) 350

Subtract 1, 7, 19… and so on.

$$350 - 1 = 349$$
$$349 - 7 = 342$$
$$342 - 19 = 323$$
$$323 - 37 = 286$$
$$286 - 61 = 225$$
$$225 - 91 = 134$$
$$134 - 127 = 7$$

Here, we see, remainder is not getting 0.

So, 350 is not a perfect cube.

We have to subtract 7 from 350 to make it perfect cube.

Now, $350 - 7 = 343$

and $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$

(d) 515

Subtract 1, 7, 19… and so on.

$$515 - 1 = 514$$
$$514 - 7 = 507$$
$$507 - 19 = 488$$
$$488 - 37 = 451$$
$$451 - 61 = 390$$
$$390 - 91 = 299$$
$$299 - 127 = 172$$
$$172 - 169 = 3$$

Here, we see, remainder is not getting 0.

So, 515 is not a perfect cube.

We have to subtract 3 from 515 to make it a perfect cube.

Now, $515 - 3 = 512$

and $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$

3. (a) 121

Subtract 1, 7, 19… and so on.

$$121 - 1 = 120$$
$$120 - 7 = 113$$
$$113 - 19 = 94$$
$$94 - 37 = 57$$
$$57 - 61 = -4$$

Here, we see that we have -4 as a remainder.

So, to make 121 a perfect cube, we need to add 4 to it.

Now, $121 + 4 = 125$

and $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

(b) 340

Subtract 1, 7, 19… and so on.

$$340 - 1 = 339$$
$$339 - 7 = 332$$
$$332 - 19 = 313$$
$$313 - 37 = 276$$
$$276 - 61 = 215$$
$$215 - 91 = 124$$
$$124 - 127 = -3$$

Here, we see that, we have -3 as a remainder.

So, to make 340 a perfect cube, we need to add 3 to it.

Now, $340 + 3 = 343$

and $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$

(c) 510

Subtract 1, 7, 19… and so on.

$$510 - 1 = 509$$
$$509 - 7 = 502$$
$$502 - 19 = 483$$
$$483 - 37 = 446$$
$$446 - 61 = 385$$
$$385 - 91 = 294$$
$$294 - 127 = 167$$
$$167 - 169 = -2$$

Here, we have -2 as remainder.

So, to make 510 a perfect cube, we need to add 2 to it.

Now, $510 + 2 = 512$

and $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$

(d) 728

Subtract 1, 7, 19… and so on.

$$728 - 1 = 727$$
$$727 - 7 = 720$$
$$720 - 19 = 701$$
$$701 - 37 = 664$$
$$664 - 61 = 603$$
$$603 - 91 = 512$$
$$512 - 127 = 385$$
$$385 - 169 = 216$$
$$216 - 217 = -1$$

Here, we have -1 as a remainder.

So, to make 728 a perfect cube, we need to add 1 to it.

Now, $728 + 1 = 729$

and $\sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9} = 9$

4. (a) 36,000

Writing 36000 as a product of its prime factors, we get

$36000 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$

Grouping them into groups of three, we get

$36000 = \underline{2 \times 2 \times 2} \times 2 \times 2 \times 3 \times 3 \times \underline{5 \times 5 \times 5}$

We see that $2 \times 2 \times 3 \times 3$ are not grouped.

To make it a perfect cube 2×2 should be multiplied by 2 and 3×3 should be multiplied by 3.

So, the smallest no.

Which should be multiplied to to make 36000 a perfect cube is $(2 \times 3) = 6$.

2	36000
2	18000
2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

(b) 3,456

Writing 3456 as a product of its prime factors, we get

$3456 = 2\times 2\times 2\times 2\times 2\times 2\times 2\times 3\times 3\times 3$

Grouping them into groups of three, we get,

$3456 = \underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times 2\times \underline{3\times 3\times 3}$

2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that 2 is not grouped.

To make it a perfect cube, 2 should be multiplied by $2\times 2 = 4$.

So, the smallest number which should be multiplied to make 3456 a perfect cube is 4.

(c) 4116

Writing 4116 as a product of its prime factors, we get.

$4116 = 2\times 2\times 3\times 7\times 7\times 7$

Grouping them into groups of three, we get.

$4116 = 2\times 2\times 3\times \underline{7\times 7\times 7}$

2	4116
2	2058
3	1029
7	343
7	49
7	7
	1

We see that 2×2 and 3 are not grouped.

To make it a perfect cube 2×2 should be multiplied by 2 and 3 should be multiplied by 3×3.

So, the smallest number which should be multiplied to make 4116 a perfect cube is $(2\times 3\times 3) = 18$.

(d) 10976

Writing 10976 as a product of its prime factors, we get

$10976 = 2\times 2\times 2\times 2\times 2\times 7\times 7\times 7$

Grouping them into groups of three, we get

$10976 = \underline{2\times 2\times 2}\times 2\times 2\times \underline{7\times 7\times 7}$

2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

We see that 2×2 are not grouped.

To make it a perfect cube 2×2 should be multiplied by 2.

(e) 8232

Writing 8232 as a product of its prime factors, we get.

$8232 = 2\times 2\times 2\times 3\times 7\times 7\times 7$

Grouping them into groups of three, we get

$8232 = \underline{2\times 2\times 2}\times 3\times \underline{7\times 7\times 7}$

2	8232
2	4116
2	2058
3	1029
7	343
7	49
7	7
	1

We see that 3 is not grouped.

To make it a perfect cube, 3 should be multiplied by $3\times 3 = 9$.

So, the smallest number which should be multiplied to make 8232 a perfect cube is 9.

5. (a) 8640

Writing 8640 as a product of its prime factors, we get

$8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

Grouping them into groups of three, we get

$8460 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 5$

We see that there is only one number 5 is left ungrouped. So, the smallest number by which 8640 should be divided to make it a perfect cube is 2.

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

(b) 2662

Writing 2662 as a product of its prime factors, we get.

$2662 = 2 \times 11 \times 11 \times 11$

Grouping them into groups of three, we get.

$2662 = 2 \times \underline{11 \times 11 \times 11}$

We see that there is only one number 2 is left ungrouped. So, the smallest number by which 2662 should be divided to make it a perfect cube is 5.

2	2662
11	1331
11	121
11	11
	1

(c) 27648

Writing 27648 as a product of its prime factors, we get.

$27648 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping them into groups of three, we get.

$27648 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432

2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that there is only one number 2 is left ungrouped.

So, the smallest number by which 27648 should be divided to make it a perfect cube is 2.

(d) 77760

Writing 77760 as a product of its prime factors, we get

$77760 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5$

Grouping them into groups of three, we get.

$77760 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 3 \times 3 \times 5$

We see that there is 3, 3 and 5 are left ungrouped. So, the smallest number by which 77760 should be divided to make it a perfect cube is $(3 \times 3 \times 5) = 45$.

2	77760
2	38880
2	19440
2	9720
2	4860
2	2430
3	1215
3	405
3	135
3	45
3	15
5	5
	1

(e) 69120

Writing 69120 as a product of its prime factors, we get

$69120 = 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2 \times 3\times 3\times 3\times 5$

Grouping them into groups of three, we get.

$69120 = \underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times \underline{2\times 2\times 2} \times \underline{3\times 3\times 3}\times 5$

We see that there is only one number 5 is left, ungrouped.

So, the smallest number by which 69120 should be divided to make it a perfect cube is 5.

2	69120
2	34560
2	17280
2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

6. (a) 729

By prime factorization,

$$729 = \underline{3\times 3\times 3}\times \underline{3\times 3\times 3}$$

Hence, $\sqrt[3]{729} = \sqrt[3]{\underline{3\times 3\times 3}\times \underline{3\times 3\times 3}}$

$= 3\times 3$

$= 9$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(b) 4,096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64

2	32
2	16
3	8
3	4
3	2
	1

By prime factorization

$$4096 = 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2$$

Hence, $\sqrt[3]{4096} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times \underline{2\times 2\times 2}\times \underline{2\times 2\times 2}}$

$\sqrt[3]{4096} = 2\times 2\times 2\times 2 = 16$

(c) 5,832

By prime factorization

$$5832 = 2\times 2\times 2\times 3\times 3\times 3\times 3\times 3\times 3$$

Hence, $\sqrt[3]{5832} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{3\times 3\times 3}\times \underline{3\times 3\times 3}}$

$\sqrt[3]{5832} = 2\times 3\times 3 = 18$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(d) 15,625

By prime factorization.

$$15625 = 5\times 5\times 5\times 5\times 5\times 5$$

Hence, $\sqrt[3]{15625} = \sqrt[3]{\underline{5\times 5\times 5}\times \underline{5\times 5\times 5}}$

$= 5\times 5$

$= 25$

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(e) 27000

By prime factorization.

$$27000 = 2\times 2\times 2\times 3\times 3\times 3\times 5\times 5\times 5$$

Hence, $\sqrt[3]{27000} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{3\times 3\times 3}\times \underline{5\times 5\times 5}}$

$$\sqrt[3]{27000} = 2\times 3\times 5 = 30$$

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

(f) 39304

By prime factorization.

$$39304 = 2\times 2\times 2\times 17\times 17\times 17$$

Hence, $\sqrt[3]{39304} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{17\times 17\times 17}}$

$$= 2\times 17$$

$$= 34$$

2	39304
2	19652
2	9826
17	4913
17	289
17	17
	1

(g) 74088

By prime factorization,

$$74088 = 2\times 2\times 2\times 3\times 3\times 3\times 7\times 7\times 7$$

Hence, $\sqrt[3]{74088} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{3\times 3\times 3}\times \underline{7\times 7\times 7}}$

$$= 2\times 3\times 7$$

$$= 42$$

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

(h) 117649

By prime factorization.

$$117649 = 7\times 7\times 7\times 7\times 7\times 7$$

Hence, $\sqrt[3]{17649} = \sqrt[3]{\underline{7\times 7\times 7}\times \underline{7\times 7\times 7}}$

$$\sqrt[3]{17649} = 7\times 7$$

$$= 49$$

7	117649
7	16807
7	2401
7	343
7	49
7	7
	1

(i) 1815848

By prime factorization.

$$1815848 = 2\times 2\times 2\times 61\times 61\times 61$$

Hence, $\sqrt[3]{1815848} = \sqrt[3]{\underline{2\times 2\times 2}\times \underline{61\times 61\times 61}}$

$$= 2\times 61$$

$$= 122$$

2	1815848
2	907924
2	453962
61	226981
61	3721
61	61
	1

(j) 2197

By prime factorization.

$$2197 = 13\times 13\times 13$$

Hence, $\sqrt[3]{2197} = \sqrt{13\times 13\times 13}$

$$\sqrt[3]{2197} = 13$$

13	2197
13	169
13	13
	1

7. Volume of a cubical tank $= 2197000 \text{ m}^3$

Let the side of cubical tank $= x$ m

$\therefore$ Volume of cubical tank $= x^3$

So, $x^3 = 2197000 \text{ m}^3$

$x^3 = 10 \times 10 \times 10 \times 13 \times 13 \times 13$

$x = \sqrt[3]{\underline{10 \times 10 \times 10} \times \underline{13 \times 13 \times 13}}$

$= 10 \times 13$

$= 130$ m

Hence, the length of the side of cubical tant is 130 m.

10	2197000
10	219700
10	21970
13	2197
13	169
13	13
	1

8. Volume of cubical box $= 2460375 \text{ cm}^3$

Let the length of side of cubical box $= x$ cm

So, $x^3 = 2460375$

or $x = \sqrt[3]{2460375}$

$= \sqrt{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}}$

$= 3 \times 3 \times 3 \times 5$

$= 135$ cm

Hence, the length of the side of cubical box is 135 cm.

3	2460375
3	820125
3	273375
3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

9. (a) Number $= 314432$

Unit digit of the number $= 2$

Unit digit of the cube root of the number $= 8$

The number left after striking out the units, tens and hundreds digits of 314432 is 314.

$\because \quad 6^3 < 314 < 7^3$

The tens digit of the cube root of the number $= 6$

$\therefore \quad \sqrt[3]{314432} = 68$

(b) Number $= 857375$

Unit digit of the number $= 5$

Unit digit of the cube-root of the number $= 5$

The number left after striking out the units, tens and hundreds digits of 857375 is 857.

$\because \quad 9^3 < 857 < 10^3$

The tens digit of the cube root of the number $= 9$.

$\therefore \quad \sqrt[3]{857375} = 95$

(c) Number $= 636056$

Unit digit of the number $= 6$

Unit digit of the cube root of the number $= 6$

The number left after striking out the units, tens and hundreds digits of 636056 is 636.

$\because \quad 8^3 < 636 < 9^3$

The tens digit of the cube root of the number $= 8$

$\therefore \quad \sqrt[3]{636056} = 86$

(d) Number $= 205379$

Unit digit of the number $= 9$

Unit digit of the cube root of the number $= 9$

The number left after striking out the units, tens and hundreds digit of 205379 is 205.

$\because \quad 5^3 < 205 < 6^3$

The tens digit of the cube root of the number $= 5$

$\therefore \quad \sqrt[3]{205379} = 59$

(e) Number $= -343000$

Unit digit of the number $= 0$

Unit digit of the cube root of the number $= 0$

The number left after striking out the units, tens and hundreds digits of – 343000 is – 343.

$\because \quad 7^3 = 343$

The tens digit of the cube root of the number $= 7$

$\therefore \quad \sqrt[3]{-343000} = -70$

(f) Number $= -132651$

Unit digit of the number $= 1$

Unit digit of the cube root of the number $= 1$

The number left after striking out the units, tens and hundreds digits of – 132651 is – 132.

$\because \quad 5^3 < 132 < 6^3$

The tens digit of the cube root of the number $= 5$

$\therefore \quad \sqrt[3]{-132651} = -51$

Exercise 7.4

1. (a) $\sqrt[3]{-343}$

$= -\sqrt[3]{343}$

Writing 343 as a product of its prime factors, we get

$343 = 7 \times 7 \times 7$

$\therefore \quad -\sqrt[3]{343} = -\sqrt[3]{7 \times 7 \times 7}$

So, $\quad -\sqrt[3]{343} = -7$

7	343
7	49
7	7
	1

(b) $\sqrt[3]{-4096000}$

$= -\sqrt[3]{4096000}$

$\therefore \quad 4096000$

$= 10 \times 10 \times 10 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

So, $-\sqrt[3]{4096000}$

$= -\sqrt[3]{= \underline{10 \times 10 \times 10} \times \underline{4 \times 4 \times 4} \times \underline{4 \times 4 \times 4}}$

$= -10 \times 4 \times 4 = -160$

10	4096000
10	409600
10	40960
4	4096
4	1024
4	256

4	64
4	16
4	4
	1

(c) $\sqrt[3]{-6859}$

$= -\sqrt[3]{6859}$

19	6859
19	361
19	19
	1

Writing 6859 as a product of its prime factors, we get

$\therefore \quad 6859 = 19 \times 19 \times 19$

So, $-\sqrt[3]{6859} = -\sqrt[3]{19 \times 19 \times 19} = -(19) = -19$

(d) $\sqrt[3]{-373248} = -\sqrt[3]{373248}$

Writing 373248 as a product of its prime factors, we get.

$\therefore \quad 373248 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

So, $\quad -\sqrt[3]{373248}$

$= -\sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}}$

$= -(2 \times 2 \times 2 \times 3 \times 3) = -72$

(e) $\sqrt[3]{2.197} = \sqrt[3]{\dfrac{2197}{1000}}$

$= \dfrac{\sqrt[3]{2197}}{\sqrt[3]{1000}} = \dfrac{\sqrt[3]{13 \times 13 \times 13}}{\sqrt[3]{10 \times 10 \times 10}} = \dfrac{13}{10} = 1.3$

(f) $\sqrt[3]{-0.729}$

$= -\sqrt[3]{0.729} = -\sqrt[3]{\dfrac{729}{1000}} = \dfrac{-\sqrt[3]{729}}{\sqrt[3]{1000}} = \dfrac{-\sqrt[3]{9 \times 9 \times 9}}{\sqrt[3]{10 \times 10 \times 10}}$

$= \dfrac{-9}{10} = -0.9$

2	39304
2	19652
2	9826
17	4913
17	289
17	17
	1

(g) $\sqrt[3]{0.039304} = \sqrt[3]{\dfrac{39304}{1000000}} = \dfrac{\sqrt[3]{39304}}{\sqrt[3]{1000000}}$

$= \dfrac{\sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{17 \times 17 \times 17}}}{\sqrt[3]{\underline{10 \times 10 \times 10} \times \underline{10 \times 10 \times 10}}}$

$= \dfrac{2 \times 17}{10 \times 10} = \dfrac{34}{100} = 0.34$

(h) $\sqrt[3]{-0.005832}$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= -\sqrt[3]{0.005832} = -\sqrt[3]{\dfrac{5832}{1000000}} = \dfrac{-\sqrt[3]{5832}}{\sqrt[3]{1000000}}$

$= \dfrac{-\sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}}}{\sqrt[3]{\underline{10 \times 10 \times 10} \times \underline{10 \times 10 \times 10}}}$

$= \dfrac{-(2 \times 3 \times 3)}{10 \times 10}$

$= \dfrac{-18}{100}$

$= -0.18$

2. (a) $\sqrt[3]{8}\times\sqrt[3]{27}=\sqrt[3]{8\times 27}$

$\sqrt[3]{2\times 2\times 2}\times\sqrt[3]{3\times 3\times 3}=\sqrt[3]{216}$

$2\times 3=\sqrt[3]{6\times 6\times 6}$

$6=6$

L.H.S. = R.H.S. **(Proved)**

(b) $\sqrt[3]{125}\times\sqrt[3]{216}=\sqrt[3]{125\times 216}$

$\sqrt[3]{5\times 5\times 5}\times\sqrt[3]{6\times 6\times 6}=\sqrt[3]{27000}$

$5\times 6=\sqrt[3]{3\times 3\times 3\times 10\times 10\times 10}$

$30=3\times 10$

$30=30$

L.H.S. = R.H.S. **(Proved)**

(c) $\sqrt[3]{-512\times 343}=\sqrt[3]{-512}\times\sqrt[3]{343}$

$\sqrt[3]{-175616}=-\sqrt[3]{512}\times\sqrt[3]{343}$

$-\sqrt[3]{175616}=-\sqrt[3]{8\times 8\times 8}\times\sqrt[3]{7\times 7\times 7}$

$-\sqrt[3]{8\times 8\times 8\times 7\times 7\times 7}=-8\times 7$

$-(8\times 7)=-56$

$-56=-56$

L.H.S. = R.H.S. **(Proved)**

(d) $\sqrt[3]{-729\times(-1000)}=\sqrt[3]{-729}\times\sqrt[3]{-1{,}000}$

$\sqrt[3]{729000}=-\sqrt[3]{729}\times-\sqrt[3]{1000}$

$\sqrt[3]{9\times 9\times 9\times 10\times 10\times 10}=-\sqrt[3]{9\times 9\times 9}\times-\sqrt[3]{10\times 10\times 10}$

$9\times 10=(-9)\times(-10)$

$90=90$

L.H.S. = R.H.S. **(Proved)**

3. (a) $\sqrt[3]{8\times 1331}=\sqrt[3]{8}\times\sqrt[3]{1331}$

$=\sqrt[3]{2\times 2\times 2}\times\sqrt[3]{11\times 11\times 11}=2\times 11=22$

(b) $\sqrt[3]{-125\times 729}=\sqrt[3]{-125}\times\sqrt[3]{729}=-\sqrt[3]{125}\times\sqrt[3]{729}$

$=-\sqrt[3]{5\times 5\times 5}\times\sqrt[3]{9\times 9\times 9}=-5\times 9=-45$

(c) $\sqrt[3]{-64\times(-343)}=\sqrt[3]{-64}\times\sqrt[3]{-343}=-\sqrt[3]{64}\times-\sqrt[3]{343}$

$=-\sqrt[3]{4\times 4\times 4}\times-\sqrt[3]{7\times 7\times 7}$

$=(-4)\times(-7)=28$

(d) $\sqrt[3]{6^3\times 8^3}=\sqrt[3]{6^3}\times\sqrt[3]{8^3}=6\times 8=48$

(e) $\sqrt[3]{-729p^6}=-\sqrt[3]{729}\times\sqrt[3]{p^6}$

$=-\sqrt[3]{9\times 9\times 9}\times\sqrt[3]{\underline{p\times p\times p}\times\underline{p\times p\times p}}$

$=-(9)\times p\times p$

$=-9p^2$ or $-(3p)^2$

(f) $125\sqrt[3]{p^6} - \sqrt[3]{125p^6} = 125\sqrt[3]{\underline{p\times p\times p}\times\underline{p\times p\times p}} - \sqrt[3]{125}\times\sqrt[3]{p^6}$

$= 125\sqrt[3]{\underline{p\times p\times p}\times\underline{p\times p\times p}} - \sqrt[3]{5\times5\times5}$

$\times\sqrt[3]{\underline{p\times p\times p}\times\underline{p\times p\times p}}$

$= 125\times(p\times p) - 5\times p\times p$

$= 125p^2 - 5p^2 = 120\,p^2$

(g) $\sqrt[3]{64}+\sqrt[3]{0.001}+\sqrt[3]{0.008} = \sqrt[3]{4\times4\times4}+\sqrt[3]{\frac{1}{1000}}+\sqrt[3]{\frac{8}{1000}}$

$= 4+\frac{\sqrt[3]{1}}{\sqrt[3]{1000}}+\frac{\sqrt[3]{8}}{\sqrt[3]{1000}}$

$= 4+\frac{\sqrt[3]{1\times1\times1}}{\sqrt[3]{10\times10\times10}}+\frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{10\times10\times10}}$

$= 4+\frac{1}{10}+\frac{2}{10} = 4+0.1+0.2 = 4.3$

(h) $\sqrt[3]{1331}+\sqrt[3]{0.027}-\sqrt[3]{0.008} = \sqrt[3]{11\times11\times11}+\sqrt[3]{\frac{27}{1000}}-\sqrt[3]{\frac{8}{1000}}$

$= \sqrt[3]{11\times11\times11}+\frac{\sqrt[3]{27}}{\sqrt[3]{1000}}-\frac{\sqrt[3]{8}}{\sqrt[3]{1000}}$

$= 11+\frac{\sqrt[3]{3\times3\times3}}{\sqrt[3]{10\times10\times10}}-\frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{10\times10\times10}}$

$= 11+\frac{3}{10}-\frac{2}{10} = +0.3-0.2$

$= 11+0.1 = 11.1$

(i) $\sqrt[3]{\frac{125}{512}}\times\frac{8}{5} = \frac{\sqrt[3]{125}}{\sqrt[3]{512}}\times\frac{8}{5} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{8\times8\times8}}\times\frac{8}{5} = \frac{5}{8}\times\frac{8}{5} = 1$

(j) $\sqrt[3]{\frac{0.729}{4.096}}\div\sqrt{\frac{0.09}{0.16}} = \sqrt[3]{\frac{729}{4096}}\div\sqrt{\frac{9}{16}} = \frac{\sqrt[3]{729}}{\sqrt[3]{4096}}\div\frac{\sqrt{9}}{\sqrt{16}}$

$= \frac{\sqrt[3]{9\times9\times9}}{\sqrt[3]{16\times16\times16}}\div\frac{\sqrt{3\times3}}{\sqrt{4\times4}} = \frac{9}{16}\div\frac{3}{4}$

$= \frac{9}{16}\times\frac{4}{3} = \frac{3}{4}$

4. (a) $\sqrt[3]{\frac{729}{2197}} = \frac{\sqrt[3]{729}}{\sqrt[3]{2197}} = \frac{\sqrt[3]{9\times9\times9}}{\sqrt[3]{13\times13\times13}} = \frac{9}{13}$

(b) $\sqrt[3]{\frac{-3375}{4913}} = \frac{\sqrt[3]{-3375}}{\sqrt[3]{4913}} = \frac{-\sqrt[3]{3375}}{\sqrt[3]{4913}} = \frac{-\sqrt[3]{15\times15\times15}}{\sqrt[3]{17\times17\times17}} = \frac{-15}{17}$

(c) $\sqrt[3]{\frac{24389}{19683}} = \frac{\sqrt[3]{24389}}{\sqrt[3]{19683}} = \frac{\sqrt[3]{29\times29\times29}}{\sqrt[3]{27\times27\times27}} = \frac{29}{27}$

(d) $\sqrt[3]{\dfrac{9261}{-1000}} = \dfrac{\sqrt[3]{9261}}{\sqrt[3]{-1000}} = \dfrac{\sqrt[3]{9261}}{-\sqrt[3]{1000}} = \dfrac{-\sqrt[3]{21\times 21\times 21}}{\sqrt[3]{10\times 10\times 10}}$

$= \dfrac{-21}{10} = -\ 2.1$

(e) $\sqrt[3]{\dfrac{-21952}{6859}} = \dfrac{\sqrt[3]{-21952}}{\sqrt[3]{6859}} = \dfrac{-\sqrt[3]{21952}}{\sqrt[3]{6859}} = \dfrac{-\sqrt[3]{28\times 28\times 28}}{\sqrt[3]{19\times 19\times 19}} = \dfrac{-28}{19}$

5. $-6125 = (-1)\times 6125$

5	6125
5	1225
5	245
7	49
7	7
	1

Writing 6125 as a product of its prime factors, we get

$6125 = 5\times 5\times 5\times 7\times 7$

Grouping them into groups of three, we get.

$6125 = \underline{5\times 5\times 5}\times 7\times 7$

We see that 7×7 is not grouped

To make it a perfect cube 7×7 should be multiplied by 7.

So, $-6125\times 7 = -42875$

or $-6125\times(-7) = 42875$

Hence, the smallest number by which – 6125 should be multiply to make it a perfect cube is 7 or – 7.

6. $-250 = (-1)\times 250$

2	250
5	125
5	25
5	5
	1

Writing 250 as a product of its prime factors, we get

$250 = 2\times 5\times 5\times 5$

Grouping them into groups of three, we get

$250 = 2\times \overline{5\times 5\times 5}$

We see that there is only one number 2 which is left ungrouped so, if we divide 250 by 2, it becomes a perfect cube.

So, $-250\div 2 = -125$

or $-250\div(-2) = 125$

Hence, the smallest number by which – 250 should be divided to make it a perfect cube is 2 or – 2.

7. Volume of cuboidal box $= 35.937\ m^3$

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

Let side of the box $= x$ m

So, $x^3 = 35.937$

$\Rightarrow\quad x^3 = \dfrac{35937}{1000}$

$x = \sqrt[3]{\dfrac{35937}{1000}}$

$x = \dfrac{\sqrt[3]{35937}}{\sqrt[3]{1000}} = \dfrac{\sqrt[3]{\underline{3\times 3\times 3}\times\underline{11\times 11\times 11}}}{\sqrt[3]{10\times 10\times 10}}$

$= \dfrac{3\times 11}{10} = \dfrac{33}{10} = 3.3$ m

Hence, the side of the cubical box is 3.3 m.

8. Volume of cube $= \frac{2197}{729}$ cm^3

Let side of cube $= x$ cm

So, volume $= x^3$ cm^3

$\therefore \quad x^3 = \frac{2197}{729} \quad \Rightarrow \quad x = \sqrt[3]{\frac{2197}{729}}$

$x = \frac{\sqrt[3]{2197}}{\sqrt[3]{729}} \quad \Rightarrow \quad x = \frac{\sqrt[3]{13 \times 13 \times 13}}{\sqrt[3]{9 \times 9 \times 9}}$

$x = \frac{13}{9}$ cm.

Hence, the side of the box is $\frac{13}{9}$ cm.

9. Let the three no. are $3x, 4x$ and $5x$.

$\because$ Sum of their cubes = 0.001728

Then, $(3x)^3 + (4x)^3 + (5x)^3 = 0.001728$

$27x^3 + 64x^3 + 125x^3 = \frac{1728}{1000000}$

$216x^3 = \frac{1728}{1000000} \quad \Rightarrow \quad x^3 = \frac{1728}{216 \times 1000000}$

$x^3 = \frac{8}{1000000} \quad \Rightarrow \quad x = \sqrt[3]{\frac{8}{1000000}}$

$x = \frac{\sqrt[3]{8}}{\sqrt[3]{1000000}} = \frac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{100 \times 100 \times 100}}$

$x = \frac{2}{100} = 0.02$

So, the numbers are : $3x = 3 \times 0.02 = 0.06$

$4x = 4 \times 0.02 = 0.08$

And, $5x = 5 \times 0.02 = 0.10$

10. Let three numbers are $2x, 3x$ and $5x$

$\because$ Sum of their cubes $= 54880$

Then, $(2x)^3 + (3x)^3 + (5x)^3 = 54880$

$8x^3 + 27x^3 + 125x^3 = 54880$

$160x^3 = 54880$

$x^3 = \frac{54880}{160}$

$x^3 = 343$

$x = \sqrt[3]{343}$

$x = \sqrt[3]{7 \times 7 \times 7}$

$x = 7$

So, the numbers are : $2x = 2 \times 7 = 14$

$3x = 3 \times 7 = 21$

And, $5x = 5 \times 7 = 35$

11. Let the numbers are $2x$ and $3x$

$\because$ Difference of their cubes = 9728.

Then, $$(3x)^3 - (2x)^3 = 9728$$
$$27x^3 - 8x^3 = 9728$$
$$19x^3 = 9728$$
$$x^3 = \frac{9728}{19}$$
$$x^3 = 512$$
$$x = \sqrt[3]{512}$$
$$x = \sqrt[3]{8 \times 8 \times 8}$$
$$x = 8$$

So, the numbers are : $2x = 2 \times 8 = 16$

And, $3x = 3 \times 8 = 24$

Multiple Choice Questions

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (b) **6.** (c) **7.** (a) **8.** (c)

Brain Teaser

I. Write 'T' for 'True' and 'F' for 'False' :

1. True 2. True 3. False 4. False 5. True 6. True

8 Comparing Quantities

Exercise 8.1

1. (a) Express $4\frac{1}{5}\%$ as a fraction.

$$4\frac{1}{5}\% = \frac{21}{5}\% = \frac{21}{5 \times 100} = \frac{21}{500}$$

(b) Express $5\frac{1}{8}$ as a per cent.

$$5\frac{1}{8} = \frac{41}{8} = \frac{41}{8} \times 100\% = \frac{41}{2} \times 25\% = \frac{1025}{2}\% = 512\frac{1}{2}\%$$

(c) Express 72 : 8 as a per cent.

$$72:8 = \frac{72}{8} = \frac{9}{1} = \frac{9}{1} \times 100\% = 900\%$$

(d) Express 37% as a decimal.

$$37\% = \frac{37}{100} = 0.37$$

2. (a) 10 of 40?

Let, x% of 40 is 10.

Then, $x = \frac{10 \times 100}{40} = 25\%$

Hence, 25% of 40 is 10.

(b) 4 of 2?

Let, x% of 2 is 4

Then, $x = \frac{4 \times 100}{2} = 200\%$

Hence, 200% of 2 is 4.

(c) 300 g of 2 kg?

Let, x% of 2 kg is 300 g

Then, $x = \frac{300g \times 100}{2\,\text{kg}}$

$= \frac{300 \times 100\,\text{g}}{2000\,\text{g}}\ \%$

$= 15\%$

Hence, 15% of 2 kg is 300g.

(d) ₹ 7.50 of ₹ 6?

Let x% of ₹ 6 is ₹ 7.50

Then, $x = \frac{₹\,7.50 \times ₹\,100}{₹\,6}$

$= \frac{750}{6}$

$= 125\%$

Hence, 125% of ` 6 is ` 7.50.

(e) $\frac{1}{3}$ of $\frac{2}{9}$?

Let, x% of $\frac{2}{9}$ is $\frac{1}{3}$

Then, $x = \frac{\frac{1}{3} \times 100}{2/9}$

$= \frac{1 \times 9 \times 100}{3 \times 2} = \frac{900}{6}$

$= 150\%$

Hence, 150% of $\frac{2}{9}$ is $\frac{1}{3}$.

(f) $\frac{1}{10}$ of $\frac{1}{8}$?

Let, x% of $\frac{1}{8}$ is $\frac{1}{10}$

Then, $= \frac{\frac{1}{10} \times 100}{1/8}$

$x = \frac{8 \times 1}{10 \times 1} \times 100 = \frac{800}{10}$

$= 80\%$

Hence, 80% of $\frac{1}{8}$ is $\frac{1}{10}$.

3. (a) $\frac{1}{20} = \frac{x}{100} = y\%$

Taking first two ratio, we get

$$\frac{1}{20} = \frac{x}{100}$$

$$20x = 100 \qquad \text{(By cross multiplication method)}$$

$$x = 100 \div 20$$

$$x = 5$$

Taking first and last ratio, we get

$$\frac{1}{20} = y\%$$

$$\frac{1}{20} = \frac{y}{100}$$

$$20\,y = 100$$

$$y = \frac{100}{20} \qquad \text{(By cross multiplication method)}$$

$$y = 5$$

Hence, $x = 5$ and $y = 5$.

(b) $$\frac{4}{5} = \frac{x}{100} = y\%$$

Taking first two ratio, we get

$$\frac{4}{5} = \frac{x}{100}$$

$$5x = 4 \times 100 \quad \text{(By cross multiplication method)}$$

$$x = 400 \div 5$$

$$x = 80$$

Taking first and last two ratio, we get

$$\frac{4}{5} = y\%$$

$$\frac{4}{5} = \frac{y}{100}$$

$$5y = 4 \times 100 \text{ (By cross multiplication method)}$$

$$y = 400 \div 5$$

$$y = 80$$

Hence, $x = 80$ and $y = 80$.

4. Let the maximum marks be x.

Thus, $$82\% \text{ of } x = 410$$

$$\frac{82}{100} \times x = 410$$

$$x = \frac{410 \times 100}{82}$$

$$x = 500$$

Hence, the maximum marks is 500.

5. Let, the original number of dolls be x.

Thus, Number of defective dolls $= 20\%$ of x

$$= \frac{20}{100} \times x = \frac{x}{5}$$

$\therefore$ Remaining dolls $= x - \frac{x}{5} = \frac{5x - x}{5} = \frac{4}{5}x$

Now, Number of damaged dolls $= 25\%$ of $\frac{4}{5}x$

$$= \frac{25}{100} \times \frac{4}{5}x$$

$$= \frac{1}{4} \times \frac{4}{5}x = \frac{x}{5}$$

$\because$ The number of left over dolls $= 4800$ (given)

$\therefore$ $$\frac{4}{5}x - \frac{x}{5} = 4800$$

$$\frac{4x - x}{5} = 4800$$

$$\frac{3x}{5} = 4800$$

$$x = \frac{4800 \times 5}{3} = 1600 \times 5 \Rightarrow 8000$$

Hence, the original number of dolls is 8000.

6. Let, the man had ₹ x.

Thus, his loss $= 25\%$ of ₹ x

$$= \frac{25}{100} \times ₹\ x = ₹\ \frac{x}{4}$$

$\because$ Money left with him = ₹ 18,000

$\therefore$ $$x - \frac{x}{4} = ₹\ 18,000$$

$$\frac{4x - x}{4} = ₹\ 18,000$$

$$\frac{3x}{4} = ₹\ 18,000$$

$$x = ₹\ \frac{18000 \times 4}{3} = ₹\ 24,000$$

Hence, the man had ₹ 24,000 originally.

7. Ravi got total marks in each subject

$= 85 + 70 + 90 + 99 + 75 = 419$

The total maximum number of marks in each subject

$= 100 + 100 + 100 + 100 + 100 = 500$

Percentage of marks Ravi got $= \left(\frac{419}{500} \times 100\right)\% = 83.8\%$

8. Let x be the number of days on which the school was open.

So, according to the questions,

$$80\% \text{ of } x = 260$$

$$\frac{80}{100} \times x = 260$$

$$x = \frac{260 \times 100}{80} = \frac{2600}{8} = 325$$

Hence, the school was open 325 days in the year.

9. Let, the monthly income of this person be x.

Thus, 15% of $x =$ ₹ 2400

$$x \times \frac{15}{100} = ₹\ 2400$$

$$x = ₹\ \frac{2400 \times 100}{15} = ₹\ 16000$$

Hence, his monthly income is ₹ 16000.

10. Let x be the mass of nickel.

Percentage of nickel $= (100 - 72 - 24)\% = 4\%$

So,mass of nickel in 5 kg of alloy = 4% of 5kg

$\therefore \quad x = \frac{4}{100} \times 5000$ g

$x = 4 \times 50$ g $= 200$ g

Hence, is the mass of nickel in 5 kg of the alloy is 200 gm.

11. Suppose Mukesh's income = ₹ 100

$\because$ 27% of 100 $= \frac{27}{100} \times 100 =$ ₹ 27

$\therefore$ Suresh's income = ₹ (100 − 27) = ₹ 73

$\therefore$ Mukesh's income is ₹ 27 more than Suresh's income

Now, percentage $= \frac{27}{73} \times 100\%$

$= \frac{2700}{73} = 36\frac{72}{73}\%$

Hence, Mukesh's income is $36\frac{72}{73}\%$ more than that of Suresh.

12. Let the original price of the car be ₹ x.

The value of car after first year $= x - 20\%$ of x.

$= x - \frac{20x}{100}$

$= \frac{10x - 2x}{10} = \frac{8}{10}x = \frac{4}{5}x$

And, the value of car after second year $= \frac{4}{5}x - 20\%$ of $\frac{4}{5}x$

$= \frac{4}{5}x - \frac{4}{5}x \times \frac{20}{100}$

$= \frac{4}{5}x - \frac{4}{5}x \times \frac{1}{5}$

$= \frac{4x}{5} - \frac{4x}{25} = \frac{20x - 4x}{25} = \frac{16}{25}x$

So, $\frac{16}{25}x =$ ₹ 420000

$x =$ ₹ $\frac{420000 \times 25}{16} =$ ₹ 656250

Hence, the original price of the car is ₹ 656250.

13. Mrs. Suchita earns per month = ₹ 11000

Percentage of her saving amount = (100 − 85)% = 15%

So, the saving amount = 15% of ₹ 11000

= ₹ 11000$\times \frac{15}{100}$ = ₹ 1650

After increment, her salary = ₹ 11000 + 20% of ` 11000

= ₹ 11000 + ₹ 11000 $\times \frac{20}{100}$

$= ₹\ 11000 + ₹\ 2200$
$= ₹\ 13200$

Now, the saving percentage $= (100 - 87)\% = 13\%$

So,the saving amount $= ₹\ 13200$ of 13%

$$= ₹\ 13200 \times \frac{13}{100} = ₹\ 132 \times 13 = ₹\ 1716$$

So, the difference between in her to both saving

$= ₹\ 1716 - 1650 = ₹\ 66$

Hence, Mrs Suchita saves ₹ 66 more now.

14. Let x be the total number of passengers before station A.

So, the number of passengers got down at station

$$A = x - 30\% \text{ of } x = x - \frac{30x}{100}$$

$$= \frac{100x - 30x}{100} = \frac{70}{100}x = \frac{7}{10}x$$

Now, the number of passengers got down at station B.

$$= \frac{7}{10}x - 70\% \text{ of } \frac{7}{10}x = \frac{7}{10}x - \frac{70}{100} \times \frac{7}{10}x$$

$$= \frac{7}{10}x - \frac{49}{100}x = \frac{70x - 49x}{100} = \frac{21}{100}x$$

So, according to the question,

$$\frac{21}{100}x = 1050$$

$$x = \frac{1050 \times 100}{21}$$

$$x = 50 \times 100$$

$$x = 5000$$

Hence, there were 5000 passengers in the train before station A.

15. The present annual income of man $= ₹\ 880000$

Increasing percentage every year $= 10\%$

(a) The income of the man after one year

$= ₹\ 880000 + 10\%$ of $₹\ 880000$

$$= ₹\ 880000 + ₹\ 880000 \times \frac{10}{100} = ₹\ 968000$$

Now, the income of the man after 2 years

$= ₹\ 968000 + 10\%$ of $₹\ 968000$

$$= ₹\ 968000 + ₹\ 968000 \times \frac{10}{100}$$

$= ₹\ 968000 + ₹\ 96800 = ₹\ 1064800$

Hence, his income will be ₹ 1064800 2 years later.

(b) Let us suppose his salary before 1 year was ₹ x.

Increase in salary is 10% of $x = \frac{10 \times x}{100} = \frac{x}{10}$

Salary after increase $= ₹\left(x+\frac{x}{10}\right) = ₹\ \frac{11}{10}x$

Thus, $\frac{11}{10}x = ₹\ 880000$

$x = ₹\frac{880000\times 10}{11} = ₹\ 800000$

Hence, his income was ₹ 800000 1 year earlier.

Exercise 8.2

1. (a) S.P. = ₹ 777.60, gain = 8%, C.P. = ?

We know that,

$$\text{C.P.} = \frac{\text{S.P.}\times 100}{100+\text{gain}} = ₹\frac{777.60\times 100}{100+8} = ₹\frac{77760}{108} = ₹\ 720$$

(b) S.P. = ₹ 261, loss = 10%, C.P. = ?

We know that,

$$\text{C.P.} = \frac{\text{S.P.}\times 100}{100-\text{loss}} = ₹\ \frac{261\times 100}{100-10} = ₹\frac{26100}{90} = ₹\ 290$$

(c) C.P. = ₹ 875, loss = 12%, S.P. = ?

We know that,

$$\text{S.P.} = \frac{(100-\text{loss}\ \%)}{100}\times \text{C.P.}$$

$$\text{S.P.} = ₹\frac{(100-12)}{100}\times 875 = ₹\ \frac{77000}{100} = ₹\ 770$$

(d) C.P. = ₹ 300, gain = 2%, S.P. = ?

We know that,

$$\text{S.P.} = \frac{(100+\text{gain}\ \%)}{100}\times \text{C.P.}$$

$$\text{S.P.} = ₹\frac{(100+2)}{100}\times 300 = ₹\ \frac{102}{100}\times 300 = ₹\ 306$$

2. (a) C.P. = ₹ 800, S.P. = ₹ 900

Since, C.P. < S.P. [So, there is a profit (gain)].

$\therefore$ gain = S.P. – C.P. = ₹ (900 – 800) = ₹ 100

So, $\text{gain\%} = \frac{\text{gain}}{\text{C.P.}}\times 100 = \left(\frac{100}{800}\times 100\right)\% = 12.5\%$

(b) C.P. = ₹ 345, S.P. = ₹ 372.60

Since, C.P. < S.P. [So, there is a profit (gain)]

$\therefore$ gain = S.P. – C.P. = ₹ (372.60 – 345) = ₹ 27.60

So, $\text{gain\%} = \frac{\text{gain}}{\text{C.P.}}\times 100\% = \left(\frac{27.60}{345}\times 10\right)\% = 8\%$

(c) C.P. = ₹ 80, S.P. = ₹ 76.80,

Since, C.P. > S.P. [So, there is a loss.]

$\therefore$ loss = C.P. – S.P. = ₹ (80 – 76.80) = ₹ 3.2

So, loss % $= \frac{\text{loss}}{\text{C.P.}} \times 100\% = \left(\frac{3.2}{80} \times 100\right)\% = 4\%$

(d) C.P. = ₹ 2.50, S.P. = ₹ 3

Since, C.P. < S.P. [So, there is a profit (gain)].

∴ gain = S.P. – C.P. = ₹ (3 – 2.50) = ₹ 0.50

So, gain% $= \frac{\text{gain}}{\text{C.P}} \times 100\% = \left(\frac{0.50}{2.50} \times 100\right)\% = 20\%$

(e) C.P. = ₹ 3.50, S.P. = ₹ 2

Since, S.P. < C.P. [So, there is a loss].

∴ loss = C.P. – S.P. = ₹ (3.50 – 2) = ₹ 1.50

So, loss% $= \frac{\text{loss}}{\text{C.P.}} \times 100\% = \left(\frac{1.50}{3.50} \times 100\right)\% = \frac{300}{7}\% = 42\frac{6}{7}\%$

(f) C.P. = ₹ 250, overhead expenses = ₹ 50 and loss = ₹ 50.

loss % $= \frac{\text{loss}}{\text{C.P.} + \text{O.E.}} = \left(\frac{50}{250+50} \times 100\right)\% = \left(\frac{5000}{300}\right)\% = 16\frac{2}{3}\%$

(g) C.P. = ₹ 500, overhead expenses = ₹ 80 and S.P. = ₹ 600

Total C.P. = ₹ (500 + 80) = ₹ 580

Since, S.P. > C.P. [So, there is a profit (gain)].

gain = S.P. – C.P. = ₹ (600 – 580) = ₹ 20

So, gain % $= \frac{\text{gain}}{\text{C.P.}} \times 100\% = \left(\frac{20}{580} \times 100\right)\% = 3.45\%$

3. C.P. of 1 dozen (i.e., 12) eggs = ₹ 16

∴ C.P. of 1 egg = ₹ $\frac{16}{12}$ = ₹ $\frac{4}{3}$

And, S.P. of 10 eggs = ₹ 18

∴ S.P. of 1 egg = ₹ $\frac{18}{10}$ = ₹ $\frac{9}{5}$

∵ C.P. < S.P. [So, their is a profit (gain)]

∴ Gain = S.P. – C.P. = ₹ $\frac{9}{5}$ – ₹ $\frac{4}{3}$ = ₹ $\left(\frac{27-20}{15}\right)$ = ₹ $\frac{7}{15}$

So, Vendor's gain percent $= \frac{\frac{7}{15}}{\frac{4}{3}} \times 100\% = \frac{7 \times 3}{15 \times 4} \times 100\% \Rightarrow 35\%$

4. A man sold a fridge (S.P.) = ₹ 6251

Let the cost price (C.P.) of the fridge be ₹ x.

Then, loss $= \frac{1}{20}$ of C.P. $= \frac{x}{20}$

∵ loss = C.P. – S.P.

∴ $\frac{x}{20} = x -$ ₹ 6251

$$x - \frac{x}{20} = ₹\ 6251$$

$$\frac{20x - x}{20} = ₹\ 6251$$

$$\frac{19}{20}x = ₹\ 6251$$

$$x = ₹\ \frac{6251 \times 20}{19}$$

$$x = ₹\ \frac{125020}{19}$$

$$x = ₹\ 6580$$

Hence, the C.P. of the fridge is ₹ 6580.

5. Hari purchased an old bike (C.P.) = ₹ 73500
Overhead expenses = ₹ 10300 + ₹ 2600
So, total C.P. = ₹(73500 + 10300 + 2600) = ₹ 86400
The selling price = ₹ 84240
Since, C.P. > S.P.
So, there is a loss.
∴ loss = C.P. – S.P. = ₹ (86400 – 84240) = ₹ 2160

So, his loss% $= \frac{\text{loss}}{\text{C.P.}} \times 100 = \left(\frac{2160}{86400} \times 100\right)\% = 2.5\%$

6. C.P. of 75 kg of apples = ₹ 30 × 75 = ₹ 2250
Overall profit% = 10%

∴ Overall profit = 10% of ₹ 2250 = ₹ $\left(\frac{10}{100} \times 2250\right)$ = ₹ 225

∴ S.P. of 75 kg o apples = ₹ 2250 + ₹ 225 = ₹ 2475
∵ Loss of one-third of apples = 5%

And, C.P. of one-third of apples $= \frac{1}{3} \times$ ₹ 2250 = ₹ 750

∴ S.P. of one-third of apples = ₹ 750 – 5% of ₹ 750

= ₹ 750 – $\frac{5}{100} \times$ ₹ 750

= ₹ 750 – ₹ 37.50 = ₹ 712.50

Now, remaining apples = 75 kg – 25 kg ⇒ 50 kg
And, S.P. of remaining apples = ₹ 2475 – ₹ 712.50 = ₹ 1762.50

∴ S.P. of 1 kg of remaining apples = ₹ $\frac{1762.50}{50}$ = ₹ 35.25

Hence, he should sell the remaining apples at the rate of ₹ 35.25 per kg.

7. C.P. of 5 clocks = ₹ 4050
Overhead expenses = ₹ 50
So, total C.P. of 5 clocks = ₹(4050 + 50) = ₹ 4100
gain% = 15%

So, the selling price of 5 clocks $= \frac{(100+\text{gain\%})}{100} \times$ C.P.

$= ₹\ \frac{(100+15)}{100} \times 4100$

$= ₹\ 115 \times 41 = ₹\ 4715$

Hence, the selling price of a clock $= ₹\ (4715 \div 5) = ₹\ 943$

8. C.P. of 20 kg of rice $= ₹\ 18 \times 20 = ₹\ 360$

C.P. of 25 kg of rice $= ₹\ 25 \times 16 = ₹\ 400$

$\therefore$ Total quantity of rice $= (20+25)$ kg $= 45$ kg

And, C.P. of the total rice $= ₹\ (360+400) = ₹\ 760$

S.P. of total rice $= ₹\ 45 \times 19 = ₹\ 855$

Since, C.P. < S.P. [So, there is a profit (gain)].

$\therefore$ Gain = S.P. − C.P. $= ₹\ (855-760) = ₹\ 95$

So, Ajay's Gain% $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \left(\frac{95}{760} \times 100\right)\% = 12.5\%$

9. Let, two varieties of rice be $5x$ and $2x$ kgs respectively.

C.P. of $5x$ kg of rice $= ₹\ 250 \times 5x = ₹\ 1250x$

C.P. of $2x$ kg of rice $= ₹\ 75 \times 2x = ₹\ 150x$

$\therefore$ Total quantity of rice $= (5x+2x)$ kg $= 7x$ kg

C.P. of total rice $= ₹\ (1250x+150x) = ₹\ 1400x$

And, S.P. of total rice $= ₹\ 7x \times 230 = ₹\ 1610x$

$\therefore$ Gain = S.P. − C.P. $= ₹(1610x-1400x) = ₹\ 210x$

So, Gain% $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \left(\frac{210x}{1400x} \times 100\right)\% = 15\%$

10. C.P. of oranges per dozen $= ₹\ 8.50$

$\therefore$ C.P. of a orange $= ₹\ \frac{8.50}{12}$

Overhead expenses per dozen $= ₹\ 0.50$

Overhead expenses per orange $= ₹\ \frac{0.50}{12}$

So, total C.P. of per hundred oranges $= ₹\left(\frac{8.50}{12} + \frac{0.50}{12}\right) \times 100$

$= ₹\left(\frac{850}{12} + \frac{50}{12}\right) = ₹\frac{900}{12} = ₹\ 75$

Gain % $= 12\%$

So, S.P. of hundred oranges

$= \frac{100+\text{gain\%}}{100} \times$ C.P. of per hundred oranges

$= ₹\ \frac{100+12}{100} \times 75 = ₹\ \frac{112}{100} \times 75 = ₹\ 84$

11. S.P. of a bucket $= ₹\ 67.50$

loss% $= 10\%$

C.P. of a bucket = ?

$\therefore$ C.P. of a bucket $= \frac{\text{S.P.} \times 100}{100 - \text{loss\%}} = ₹\ \frac{67.50 \times 100}{100-10}$

$= ₹\ \frac{6750}{90} = ₹\ 75$

If the new S.P. be ₹ 82.50

Then, there is a profit. ($\because$ S.P. > C.P.)

($\therefore$ Profit = ₹ 82.50 – ₹ 75 = ₹ 7.5)

So, profit% $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \left(\frac{7.5}{75} \times 100\right)\% = 10\%$

12. Let the original S.P. be ₹ x

If, profit % = 5%

Thus, C.P. $= \frac{\text{S.P.} \times 100}{100 + \text{gain\%}} = ₹\ \frac{x \times 100}{100+5} = ₹\ \frac{100}{105}x = ₹\ \frac{20}{21}x$

By reducing the selling price of an article by ₹ 50.

So, new S.P. = ₹ $(x - 50)$

And, loss% = 5%

Thus, C.P. of the article $= \frac{\text{New S.P.} \times 100}{100 - \text{loss\%}}$

$$\frac{20}{21}x = ₹\ \frac{(x-50) \times 100}{100-5}$$

$$\frac{20}{21}x = ₹\ \frac{100x - 5000}{95}$$

$$2 \times 95x = ₹\ (10x - 500) \times 21$$

$$190x = 210x - ₹\ 10500$$

$$210x - 190x = ₹\ 10500$$

$$20x = ₹\ 10500$$

$$x = ₹\ (10500 \div 20)$$

$$x = ₹\ 525$$

Hence, the original selling price of the article is ₹ 525.

13. S.P. of oranges per dozen = ₹ 72

$\therefore$ S.P. of a orange = ₹ $72 \div 12$ = ₹ 6

And, S.P. of 100 oranges = ₹ 6×100 = ₹ 600

If she loses = 10%

Thus, C.P. of 100 oranges $= \frac{\text{S.P. of 100 oranges} \times 100}{100 - \text{loos\%}}$

$= ₹\ \frac{600 \times 100}{(100-10)} = ₹\ \frac{60000}{90} = ₹\ \frac{2000}{3}$

$\because$ New, S.P. of 100 oranges = ₹ 600

Since, S.P. < C.P. [So, there is a loss].

$\therefore$ loss C.P. – S.P. $= ₹\left(\frac{2000}{3} - 600\right) = ₹\ \frac{200}{3}$

So, her loss% $= \frac{100 \times \text{loss}}{\text{C.P. of 100 oranges}}$

$$= ₹\frac{100 \times \frac{200}{3}}{\frac{2000}{3}} = ₹\ \frac{\frac{20000}{3}}{\frac{2000}{3}} = 10\%$$

14. Let x be the C.P. of an article for Dev.

profit of Dev $= 20\%$

So, S.P. of an article for Dev $= \frac{100 + \text{gain\%}}{100} \times$ C.P. $= ₹\frac{100+20}{100} \times x$

$$= ₹\ \frac{12}{10}x = ₹\ \frac{6}{5}x$$

This S.P. will be C.P. for Radha.

So, now C.P. for Radha $= ₹\ \frac{6}{5}x$

And profit of Radha $= 4\%$

So, S.P. of an article for Radha $= \frac{100 + \text{gain\%}}{100} \times$ C.P.

$$= ₹\ \frac{100+4}{100} \times \frac{6}{5}x$$

$$= \frac{104}{100} \times \frac{6}{5}x \text{ GST} = \frac{26}{25} \times \frac{6}{5}x = \frac{156}{125}x$$

This S.P. will be C.P. for Jack. But Jack has paid ₹ 624.

So, $\frac{156}{125}x = ₹\ 624$

$$x = ₹\ \frac{624 \times 125}{156}$$

$$x = ₹\ \frac{78000}{156}$$

$$x = ₹\ 500$$

Hence, the cost price of the given article for Dev is ₹ 500.

15. Let the cost price of each candle be ₹ 1.

So, the C.P. of 12 candles = ₹ 12

And, C.P. of 15 candles = ₹ 15

But, the C.P. of 12 candles = S.P. of 15 candles

$\therefore$ the S.P. of 15 candles = ₹ 12

And the S.P. of 12 candles = ₹ 12

$\because$ S.P. < C.P.

So, their is a loss

$\therefore$ loss = C.P. – S.P. $= ₹\ (15 - 12) = ₹\ 3$

So, $\text{loss\%} = \frac{\text{loss}}{\text{C.P.}} \times 100 = \left(\frac{3}{15} \times 100\right)\% = 20\%$

16. S.P. of a bouquet = ₹ 322

And, gains = 15%

$\therefore$ C.P. of a bouquet $= \frac{\text{S.P.} \times 100}{100 + \text{gain\%}}$

$$= ₹\ \frac{322 \times 100}{100 + 15} = ₹\frac{32200}{115} = ₹\ 280$$

For gain 25%, S.P. $= \frac{100 + \text{gain\%}}{100} \times \text{C.P.}$

$$= ₹\left(\frac{100 + 25}{100}\right) \times 280 = ₹\ \frac{125 \times 280}{100} = ₹\ 350$$

Hence, he should sell bouquet for ₹ 350 to get gain 25%.

17. Let the cost price (C.P.) of the toy be ₹ x.

Profit % = 12%

$\therefore$ S.P. of the toy = C.P. + gain $= x + \frac{12}{100} \times x = \frac{28}{25}x$

If it had been sold for ₹ 33 more.

Then, new S.P. of the toy $= \left(\frac{28}{25}x + ₹\ 33\right)$

Thus, profit% = 14%

So, the C.P. of the toy $= \frac{\text{New S.P.} \times 100}{100 + \text{gain\%}}$

$$x = \frac{\left(\frac{28}{25}x + 33\right) \times 100}{100 + 14}$$

$$114x = 112x + 3300$$

$$114x - 112x = 3300$$

$$2x = ₹\ 3300$$

$$x = ₹\ 3300 \div 2$$

$$x = ₹\ 1650$$

Hence, the cost price of the toy is ₹ 1650.

18. C.P. of first hockey stick = ₹ 560

And, profit % = 15%

$\therefore$ S.P. of first hockey stick $= \frac{100 + \text{profit\%}}{100} \times \text{C.P.}$

$$= ₹\ \frac{100 + 15}{100} \times 560 = ₹\ \frac{115}{100} \times 560$$

$$= ₹\ \frac{64400}{100} = ₹\ 644$$

C.P. of second hockey stick = ₹ 240

And, loss % = 5%

$\therefore$ S.P. of second hockey stick $= \frac{100 - \text{loss}\%}{100} \times$ C.P.

$= ₹\ \frac{100-5}{100} \times 240 = ₹\ \frac{95}{10} \times 24$

$= ₹\ \frac{2280}{10} = ₹\ 228$

Total C.P. of two hockey sticks = ₹(560 + 240) = ₹ 800

And, total S.P. of two hockey sticks = ₹ (644 + 228) = ₹ 872

Since, S.P. > C.P.

[So, there is a profit in the whole transaction.]

$\therefore$ Profit = S.P. – C.P. = ₹ (872 – 800) = ₹ 72

So, overall Profit % $= \left(\frac{\text{Profit}}{\text{C.P.}} \times 100\right)\% = \left(\frac{72}{800} \times 100\right)\% = 9\%$

Exercise 8.3

1. (a) M.P. = ₹ 850, Discount% = 5%, S.P. = ?

$\therefore$ Discount $= \frac{\text{Discount\%} \times \text{M.P.}}{100} = ₹\ \frac{5 \times 850}{100} = ₹\ 42.50$

So, S.P. = M.P. – Discount = ₹ 850 – ₹ 42.50 = ₹ 807.50

(b) M.P. = ₹ 1550, Discount% = 8%

$\therefore$ Discount $= \frac{\text{Discount \%} \times \text{M.P.}}{100} = ₹\ \frac{8 \times 1550}{100} = ₹\ 124$

So, S.P. = M.P. – Discount = ₹ (1550 – 124) = ₹ 1426

2. (a) S.P. = ₹ 1880, Discount% = 6%, M.P. = ?

$\therefore$ M.P. $= \frac{\text{S.P.} \times 100}{100 - \text{Discount\%}}$

$= ₹\ \frac{1880 \times 100}{100-6} = ₹\ \frac{188000}{94} = ₹\ 2000$

(b) S.P. = ₹ 2392, Discount% = 8%, M.P. = ?

$\therefore$ M.P. $= \frac{\text{S.P.} \times 100}{100 - \text{Discount\%}} = ₹\ \frac{2392 \times 100}{100-8}$

$= ₹\ \frac{239200}{92} = ₹\ 2600$

3. (a) M.P. = ₹ 800, S.P. = ₹ 750, Discount% = ?

$\therefore$ Discount = M.P. – S.P.

= ₹ 800 – ₹ 750 = ₹ 50

So, Discount % $= \frac{\text{Discount}}{\text{M.P.}} \times 100\%$

$= \frac{50}{800} \times 100\%$

$= \frac{50}{8} = 6.25\%$

(b) M.P. = ₹ 950, S.P. = ₹ 760, Discount%

∴ Discount = M.P. – S.P.

= ₹ 950 – ₹ 760 = ₹ 190

So, Discount % $= \frac{\text{Discount}}{\text{M.P.}} \times 100\%$

$= \frac{190}{950} \times 100\% = \frac{200}{5}\% = 20\%$

4. Discount% = 35%, M.P. of shirt = ₹ 500, S.P. = ?

∴ S.P. of shirt = M.P. – Discount

$= \text{M.P.} - \frac{\text{Discount\%} \times \text{M.P.}}{100}$

$= ₹\ 500 - ₹\ \frac{35 \times 500}{100}$

= ₹ 500 – ₹ 175 = ₹ 325

Hence, I need to pay ₹ 325 for the shirt.

5. C.P. of a pack of juice = ₹ 90

∴ C.P. two pack of juices = ₹ 180

Because there is a scheme buy one get one free

So, S.P. of two pack of juices = ₹ 90

(a) Discount = C.P. – S.P. = ₹ 180 – ₹ 90 = ₹ 90

(b) Discount% $= \frac{\text{Discount}}{\text{M.P.}} \times 100 = \left(\frac{90}{180} \times 100\right)\% = 50\%$

6. M.P. of fan = ₹ 1250, Discount% = 6%, S.P. = ?

∴ S.P. of fan = M.P. – Discount

$= \text{M.P.} - \frac{\text{Discount\%} \times \text{M.P.}}{100}$

$= ₹\ 1250 - ₹\frac{6 \times 1250}{100} = ₹\ 1250 - ₹\ 75 = ₹\ 1175$

Hence, the selling price of the given fan is ₹ 1175.

7. Let the amount paid by the shopkeeper be ₹ x.

S.P. of 1 kg of sugar and a chocolate = ₹ 28

And, cost of giving chocolate = ₹ 5

∴ S.P. of 1 kg sugar = ₹ (28 – 5) = ₹ 23

And profit% = 25%

So, C.P. of 1 kg of sugar $= ₹\ \frac{\text{S.P.} \times 100}{100 + \text{Profit\%}} = ₹\ \frac{23 \times 100}{100 + 15}$

$= ₹\ \frac{2300}{115} = ₹\ 20$

Hence, the shopkeeper is ₹ 20 for 1 kg of sugar.

8. C.P. of goods = ₹ 1100

And, profit % = 20%

∴ M.P. = C.P. + 20% of C.P.

$= ₹\ 1100 + ₹\ 1100 \times \frac{20}{100}$

$= ₹\ (1100 + 220) = ₹\ 1320$

Discount = Discount% of M.P.

= 20% of M.P.

$= \frac{20}{100} \times ₹\ 1320 = ₹\ 264$

$\therefore$ S.P. = M.P. – Discount = ₹ 1320 – ₹ 264 = ₹ 1056

Since, S.P. < C.P.

So, there is a loss.

Loss = C.P. – S.P. = ₹ 1100 – 1056 = ₹ 44

So, merchant's loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100 = \left(\frac{44}{1100} \times 100\right)\% = 4\%$

9. Discount% = 25% , and Profit% = 20%

gains = ₹ 360

$$\text{Profit\%} = \frac{\text{gains}}{\text{C.P.}} \times 100$$

$$20 = \frac{360}{\text{C.P.}} \times 100$$

$$\text{C.P.} = ₹\ \frac{360 \times 100}{20}$$

C.P. = ₹ 360 × 5

C.P. = ₹ 1800

Now, S.P. = C.P. + gain

= ₹ 1800 + ₹ 360 = ₹ 2160

So, $\text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount\%}}$

$= ₹\ \frac{2160 \times 100}{100 - 25} = ₹\ \frac{2160 \times 100}{75}$

$= ₹\ \frac{2160 \times 4}{3} = ₹\ 720 \times 4 \Rightarrow ₹\ 2880$

Hence, the marked price of the cycle is ₹ 2880.

10. M.P. = ₹ 800, discount = 10%, profit% = 20%

C.P.= ? , Profit% (if no discount) = ?

$$\text{S.P.} = \text{M.P.} - \frac{\text{Discoun\%} \times \text{M.P.}}{100}$$

$= ₹\ 800 - ₹\ \frac{10 \times 800}{100} = ₹\ 800 - ₹\ 80 = ₹\ 720$

$\therefore$ $\text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{gain\%}}$

$= ₹\ \frac{720 \times 100}{100 + 20} = ₹\ \frac{72000}{120} = ₹\ 600$

Now, if no discount had been allowed.

Thus, $\qquad$ S.P. = M.P. = ₹ 800

So, new profit% $= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

$= \left(\frac{800-600}{600} \times 100\right)\%$

$= \left(\frac{200}{600} \times 100\right)\% = 33\frac{1}{3}\%$

11. M.P. of a DVD = ₹ 4500

First discount% = 10%

Selling price after 1st discount = ₹ 4500 – ₹ 4500 × $\frac{10}{100}$

= ₹ 4500 – ₹ 450 = ₹ 4050

Second discount% = 5%

Selling price after 2nd discount= ₹ 4050 – ₹ 4050 × $\frac{5}{100}$

= ₹ 4050 – ₹ 202.50 = ₹ 3847.50

Hence, the net selling price of the DVD is ₹ 3847.50.

12. M.P. of a bicycle = ₹ 1500

First discount% = 8%

Selling price after 1st discount = ₹ 1500 – ₹ 1500 × $\frac{8}{100}$

= ₹ 1500 – ₹ 120 = ₹ 1380

Second discount% = 4%

So, selling price after 2nd discount = ₹ 1380 – ₹ 1380 × $\frac{4}{100}$

= ₹ 1380 – ₹ 55.20 = ₹ 1324.80

And $\qquad$ third discount% = $2\frac{1}{2}\% = \frac{5}{2}\%$

So, selling price after third discount = ₹ 1324.80 – ₹ 1324.80 × $\frac{5}{2 \times 100}$

= ₹ 1324.80 – ₹ 33.12 = ₹ 1291.68

Hence, the net selling price of the bicycle is ₹ 1291.68

13. Let the M.P. be ₹ 100.

1st discount% = 8%

Selling price after 1st discount = ₹ 100 – ₹ 8 = ₹ 92

Second discount% = 5%

Selling price after second discount = ₹ 92 – ₹ 92 × $\frac{5}{100}$

= ₹ 92 – ₹ 4.60 = ₹ 87.40

Thus, $\qquad$ S.P. = ₹ 87.40

Total discount = ₹ (100 – 87.40) = ₹ 12.60

Rate of equivalent discount% $= \dfrac{\text{Total discount}}{\text{M.P.}} \times 100$

$= \left(\dfrac{12.60}{100} \times 100\right)\% = 12.60\%$

Hence, the single discount of 12.60% is equivalent to two successive discounts of 8% and 5%.

14. For discount 14%

Let the M.P. be ₹ 100

Single discount% $= 14\%$

So, selling price after single discount = ₹ 100 − ₹ 14 = ₹ 86

And for two successive discounts of 12% and 2%

1st discount% $= 12\%$

So, selling price after 1st discount = ₹ 100 – ₹ 12 = ₹ 88

Second discount% $= 2\%$

So, selling price after 2nd discount = ₹ 88 – ₹ 88 × $\dfrac{2}{100}$

= ₹ 88 – ₹ 1.76 = ₹ 86.24

Since, ₹ 86 < ₹ 86.24

So, the single discount of 14% is a better deal for the customer.

Exercise 8.4

1. List price of some stationary = ₹ 450

Sales tax charged is 7% = ₹$\dfrac{7 \times 450}{100}$ = ₹ 31.50

Amount = ₹ 31.50

∴ Mohan paid to buy some stationary = ₹ (450+ 31.50) = ₹ 481.50

2. Selling price of a geyser = ₹ 1242

Let list price be ₹ x

GST is 8% of ₹$x = \dfrac{8x}{100} = \dfrac{2}{25}x$

∴ $x + \dfrac{2}{25}x$ = ₹ 1242

$\dfrac{25x + 2x}{25}$ = ₹ 1242

$\dfrac{27x}{25}$ = ₹ 1242

x = ₹ $\dfrac{1242 \times 25}{27}$

x = ₹ 46 × 25 = ₹ 1150

Thus, the selling price (without GST) of the geyser is ₹ 1150.

3. List price of a music system = ₹ 7540

Sales tax charged is 7% = ₹$\dfrac{7 \times 7540}{100}$ = ₹ $\dfrac{52780}{100}$ = ₹ 527.80

Tax Amount = ₹ 527.80

So, The selling price of the music system = ₹ (7540 + 527.80)

= ₹ 8067.80

4. Let the M.P. of the mixer grinder be ₹ x.

So, selling price after discount $= x - \frac{10}{100}x = \frac{9}{10}x$

Sales tax charged is 5% = ₹ $\frac{5 \times \frac{9}{10}x}{100} = \frac{9}{200}x$

$\therefore$ Selling price with tax $= \frac{9}{10}x + \frac{9}{200}x = \frac{189}{200}x$

Selling price with tax is equal to ₹ 945

Now, $\frac{189}{200}x =$ ₹ 945

$\Rightarrow$ $x =$ ₹ $\frac{945 \times 200}{189}$

$x =$ ₹ 1000

Hence, the market price of the mixer grinder is ₹ 1000.

5. Amount of sales tax paid = ₹(1980 − 1800) = ₹ 180

$\therefore$ Rate of sales tax $= \left(\frac{180}{1800} \times 100\right)\% = 10\%$.

6. Selling price of a stereo system including GST is ₹ 13407.

Let M.P. be ₹ x.

GST is 9% of ₹$x = \frac{9 \times x}{100} = \frac{9x}{100}$

$\therefore$ $x + \frac{9x}{100} =$ ₹ 13407

$\Rightarrow$ $\frac{100x + 9x}{100} =$ ₹ 13407

$\Rightarrow$ $\frac{109x}{100} =$ ₹ 13407

$\Rightarrow$ $x =$ ₹ $\frac{13407 \times 100}{109} =$ ₹ 12300

Hence, the marked price of the stereo system is ₹ 12300.

7. Let, the marked price of pant be ₹ x and shirt be ₹ y.

$\therefore$ $x + y =$ ₹ 980 …(1)

GST on pant = 10% of M.P. = 10% of x

$= \frac{10}{100} \times x = \frac{x}{10}$

And, GST on shirt = 5% of M.P. = 5% of y

$= \frac{5}{100} \times y = \frac{y}{20}$

$\because$ Total GST = ₹ 94

$\therefore$ $$\frac{x}{10}+\frac{y}{20}=₹\ 94$$

$$\frac{2x+y}{20}=₹\ 94$$

$$2x+y=₹\ 94\times 20$$

$$2x+y=₹\ 1880$$

From equation (1)

$$2x+₹\ 980-x=₹\ 1880 \qquad \{\because y=₹\ 980-x\}$$

$$2x-x=₹\ 1880-₹\ 980$$

$$x=₹\ 900$$

Thus, $$y=₹\ 980-x=₹\ 980-₹\ 900$$

$$y=₹\ 80$$

Hence, the marked price of pant is ₹ 900 and a shirt is ₹ 80.

8. Selling price of a watch including GST is ₹ 330.

Let list price be ₹ x.

GST is 10% of ₹$x=\frac{10\times x}{100}=\frac{x}{10}$

$\therefore$ $$x+\frac{x}{10}=₹\ 330$$

$$\frac{11x}{10}=₹\ 330$$

$$x=₹\ \frac{330\times 10}{11}=₹\ 300$$

And selling price of a pen including GST is ₹ 212.

Let list price be ₹ y.

GST is 6 % of ₹$y=\frac{6\times y}{100}=\frac{3y}{50}$

$\therefore$ $$y+\frac{3y}{50}=₹\ 212=\frac{50y+3y}{50}=₹\ 212=\frac{53y}{50}=₹\ 212$$

$$y=₹\ \frac{212\times 50}{53}$$

$$y=₹\ 4\times 50=₹\ 200$$

Hence, the M.P. of watch and pen are ₹ 300 and ₹ 200 respectively.

9. **For pulse :**

List price of a pulse is ₹ 80

Sales tax charged is 5% $=₹\ \frac{5\times 80}{100}$

Tax amount $=₹\ 4$

So, selling price of pulse $=₹\ (80+4)=₹\ 84$

For tined food :

List price of a tined food is ₹ 60.

Sales tax charged is 8% $= ₹\ \frac{8 \times 60}{100}$

Tax amount = ₹ 4.80

So, selling price of tined food = ₹ (60 + 4.80) = ₹ 64.80

Since, she gave a 1000 rupee note to the shopkeeper.

Thus, the balance amount returned to her = ₹ (1000 − 84 − 64.80)

= ₹ (1000 − 148.80)

= ₹ 851.20

10. Let ₹ x be the marked price of the article.

Now, if the rate of GST increases by 2% then selling price of an article goes up by ₹ 140.

So, GST is 2% of ₹ x = ₹ 140

$$\frac{2 \times x}{100} = ₹\ 140$$

$$x = ₹\ \frac{140 \times 100}{2}$$

$$x = ₹\ 7000$$

Hence, is the marked price of the article is ₹ 7000.

Exercise 8.5

1. $P = ₹\ 18000$, $T = 3$ years, $R = 6\%$, $I = ?$

$$\text{S.I.} = \frac{P \times R \times T}{100} = ₹\ \frac{18000 \times 6 \times 3}{100} = ₹\ 180 \times 6 \times 3 = ₹\ 3240$$

Hence, Ravi will get ₹ 3240 as interest at the end of 3 years.

2. $P = ₹\ 24000$, $R_1 = 7\%$, $T = 2$ years, $R_2 = 10\%$

For Amir : $\text{S.I.} = \frac{P \times R \times T}{100} = ₹\ \frac{24000 \times 7 \times 2}{100}$

$= ₹\ 240 \times 14 = ₹\ 3360$

For Kabir : $\text{S.I.} = \frac{P \times R \times T}{100} = ₹\ \frac{24000 \times 10 \times 2}{100}$

$= ₹\ 240 \times 20 = ₹\ 4800$

Hence, Amir's gain = ₹ (4800 − 3360) = ₹ 1440

3. Let ₹ x be the value of the wrist watch.

$P = ₹\ 16000$, $R = 18\%$, $T = 2$ years and $A = ₹\ (20800 + x)$

$\because$ $I = A - P$

$\therefore$ $\text{S.I.} = \frac{P \times R \times T}{100} = (20800 + x) - 16000$

$$₹\ \frac{16000 \times 18 \times 2}{100} = ₹\ (20800 - 16000) + x$$

$$₹\ 160 \times 36 = ₹\ 4800 + x$$

$$x = ₹\ 5760 - ₹\ 4800$$

$$x = ₹\ 960$$

Hence, the value of the wrist watch is ₹ 960.

4. $P = ₹\ 100000,\ R = 12\%,\ T = 1\text{ year},\ I = ?$

$$\text{S.I.} = \frac{P \times R \times T}{100} = ₹\ \frac{100000 \times 12 \times 1}{100} = ₹\ 12000$$

The amount of 3 scholarships = ₹ 12000

So, the value of each scholarship = ₹$(12000 \div 3)$ = ₹ 4000

5. $I = ₹\ 812,\ R = 3\frac{1}{2}\% = \frac{7}{2}\%,\ T = 7\frac{1}{4}\text{ years} = \frac{29}{4}\text{ years}$

$\because$ $$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$₹\ 812 = \frac{P \times \frac{7}{2} \times \frac{29}{4}}{100}$$

$$₹\ 81200 = \frac{P \times 7 \times 29}{8}$$

$$P = ₹\ \frac{81200 \times 8}{7 \times 29}$$

$$P = ₹\ \frac{11600 \times 8}{29} = ₹\ 400 \times 8$$

$$P = ₹\ 3200$$

Hence, the principal is ₹ 3200.

6. $P = ₹\ 8450,\ T = 4\text{ years},\ A = ₹\ 12506,\ R = ?$

$\because$ $$I = A - P$$

$\therefore$ $$I = ₹\ (12506 - 8450) = ₹\ 4056$$

So, $$\text{S.I.} = \frac{P \times R \times T}{100}$$

$\therefore$ $$4056 = \frac{8450 \times R \times 4}{100}$$

$$R = \frac{4056 \times 100}{8450 \times 4}\% = \frac{405600}{33800}\%$$

$$R = \frac{24}{2}\% = 12\%$$

Hence, the rate of interest was 12% p.a.

7. $P = ₹\ 5000,\ A = ₹\ 5500,\ R = 8\%,\ T = ?$

$\because$ $$I = A - P$$

$\therefore$ $$I = ₹\ (5500 - 5000) = ₹\ 500$$

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$500 = \frac{5000 \times 8 \times T}{100}$$

$$T = \frac{500 \times 100}{5000 \times 8}$$

$$T = \frac{500}{400} = 1\frac{1}{4} \text{ years}$$

Hence, the time is $1\frac{1}{4}$ years.

8. $T = ?, R = 12\frac{1}{2}\% = \frac{25}{2}\%$

Let, $P = x$, Then, $A = 2x$

$\because \quad I = A - P$

$\therefore \quad I = 2x - x = x$

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$x = \frac{x \times 25 \times T}{2 \times 100}$$

$$1 = \frac{T}{2 \times 4}$$

$$T = 8 \text{ years}$$

Hence, the time is 8 years.

9. **Given :** $P = ₹\ 4000, R = 7\frac{1}{2}\% = \frac{15}{2}\%$

Time $= (29 + 31 + 30 + 31 + 25)$ days $= 146$ days

$$\therefore \quad \text{S.I.} = \frac{P \times R \times T}{100}$$

$$= ₹\ \frac{4000 \times \frac{15}{2} \times \frac{146}{365}}{100} \qquad \{\because 1 \text{ year} = 365 \text{ days}\}$$

$$= ₹\ 400 \times \frac{15}{2} \times \frac{2}{5}$$

$$= ₹\ 40 \times 3 \Rightarrow ₹\ 120$$

Hence, the interest paid ₹ 120.

Exercise 8.6

1. We know that, $\quad \text{C.I.} = P\left\{\left(1 + \frac{R}{100}\right)^n - 1\right\}$

(a) $P = ₹\ 24000, n = 3$ years, $R = 5\%$

$$\text{C.I.} = P \times \left\{\left(1 + \frac{R}{100}\right)^n - 1\right\}$$

$$\text{So, C.I.} = ₹\ 24000\left\{\left(1 + \frac{5}{100}\right)^3 - 1\right\} = ₹\ 24000\left\{\left(1 + \frac{1}{20}\right)^3 - 1\right\}$$

$$= ₹\ 24000\left\{\left(\frac{21}{20}\right)^3 - 1\right\} = ₹\ 24000\left\{\frac{21 \times 21 \times 21}{20 \times 20 \times 20} - 1\right\}$$

$$= ₹\ 24000\left(\frac{9261}{8000}-1\right) = ₹\ 24000\left(\frac{9261-8000}{8000}\right)$$

$$= ₹\ 24000\times\frac{1261}{8000} = ₹\ 3\times 1261 = ₹\ 3783$$

$\because$ A = C.I. + P

$\therefore$ A = ₹ (24000 + 3783) = ₹ 27783

(b) P = ₹ 50000, $n = 3$ years, $R = 12\%$

$$\text{C.I.} = P\times\left\{\left(1+\frac{R}{100}\right)^n-1\right\}$$

So, $$\text{C.I.} = ₹\ 50000\left\{\left(1+\frac{12}{100}\right)^3-1\right\} = ₹\ 50000\left\{\left(1+\frac{3}{25}\right)^3-1\right\}$$

$$= ₹\ 50000\left\{\left(\frac{28}{25}\right)^3-1\right\} = ₹\ 50000\left(\frac{28\times 28\times 28}{25\times 25\times 25}-1\right)$$

$$= ₹\ 50000\left\{\frac{21952}{15625}-1\right\} = ₹\ 50000\left(\frac{21952-15625}{15625}\right)$$

$$= ₹\ \frac{50000\times 6327}{15625} = ₹\ \frac{16\times 6327}{5}\ ₹\ 20246.4$$

$\therefore$ A = C.I.+ P= ₹ 20246.4 + 50000) = ₹ 70246.4

(c) P = ₹ 16000, $n = 2$ years, $R = 10\%$

$$\text{C.I.} = P\times\left\{\left(1+\frac{R}{100}\right)^n-1\right\}$$

So, $$\text{C.I.} = ₹\ 16000\left\{\left(1+\frac{10}{100}\right)^2-1\right\}$$

$$\text{C.I.} = ₹\ 16000\left\{\left(1+\frac{1}{10}\right)^2-1\right\} = ₹\ 16000\left\{\left(\frac{11}{10}\right)^2-1\right\}$$

$$= ₹\ 16000\left(\frac{11\times 11}{10\times 10}-1\right) = ₹\ 16000\left(\frac{121}{100}-1\right)$$

$$= ₹\ 16000\times\left(\frac{121-100}{100}\right)$$

$$= ₹\ 16000\times\frac{21}{100} = ₹\ 160\times 21 = ₹\ 3360$$

$\because$ A= C.I. + P

$\therefore$ A= ₹ (16000 + 3360) = ₹ 19360

2. Since, the interest is compounded half-yearly.

(a) P = ₹ 4000, $R = 10\%$ P.a.$= 10\% \div 2$ half yearly $= 5\%$ per unit.

$n = 1\frac{1}{2}$ year $= 3$ half year or 3 conversion periods

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n$

$= ₹\ 4000\left(1+\frac{5}{100}\right)^3 = ₹\ 4000\left(1+\frac{1}{20}\right)^3$

$= ₹\ 4000 \times \frac{21\times 21\times 21}{20\times 20\times 20}$

$= ₹\ \frac{9261}{2} = ₹\ 4630.50$

Now, C.I. $= A - P = ₹(4630.50 - 4000) = ₹\ 630.50$

(b) $P = ₹\ 2560,\ R = 12\frac{1}{2}\% = \frac{25}{2}\%$ p.a. $= \frac{25}{4}\%$ per unit

$n = 1$ year $= 2$ half year or 2 conversion period

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n = ₹\ 2560\left(1+\frac{\frac{25}{4}}{100}\right)^2$

$= ₹\ 2560\left(1+\frac{1}{16}\right)^2 = ₹\ 2560 \times \frac{17\times 17}{16\times 16}$

$= ₹10\times 17\times 17 = ₹\ 2890$

Now, C.I. $= A - P = ₹\ 2890 - ₹\ 2560 = ₹\ 380$

(c) $P = ₹\ 4096,\ R = 10\% = 5\%$ per unit

$n = 18$ months $= 3$ half year or 3 conversion periods

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n = ₹\ 4096\left(1+\frac{5}{100}\right)^3$

$= ₹\ 4096\left(1+\frac{1}{20}\right)^3$

$= ₹\ 4096 \times \frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$

$= ₹512\times\frac{9261}{1000} = ₹\ 4741.63$

Now, C.I. $= A - P$

$= ₹\ (4741.63 - 4096) = ₹\ 645.63$

(d) $P = ₹\ 12000,\ R = 6\%$ p.a. $= 3\%$ per unit

$n = 1\frac{1}{2}$ years $= 3$ half year or 3 conversion periods

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n$

$= ₹\ 12000\left(1+\frac{3}{100}\right)^3 = ₹\ 12000 \times \frac{103}{100}\times\frac{103}{100}\times\frac{103}{100}$

$= ₹\frac{13112724}{1000} = ₹\ 13112.72$

Now, C.I. $= A - P =$ ₹ (13112.72 – 12000) = ₹ 1112.72

3. Since, the interest is compounded quarterly.

(a) $P =$ ₹ 8000, $R = 16\%$ p.a. ÷ 4 = 4% per quarter.

$n = 9$ months $= \frac{9}{3}$ quarters $= 3$ quarters

$$\therefore \quad A = P\left(1+\frac{R}{100}\right)^{n\,(\text{no. of units})}$$

$$= ₹\ 8000\left(1+\frac{4}{100}\right)^3 = ₹\ 8000\left(1+\frac{1}{25}\right)^3$$

$$= ₹\ 8000 \times \frac{26\times 26\times 26}{25\times 25\times 25}$$

$$= ₹\ 320 \times \frac{17576}{25\times 25} = ₹\ \frac{64\times 17576}{125} = ₹\ 8998.91$$

Now, C.I. $= A - P =$ ₹ (8998.91 – 8000) = ₹ 998.91

(b) $P =$ ₹ 20000, $R = 16\%$ p.a. ÷ 4 = 4% per quarter.

$n = 6$ months $= \frac{6}{3}$ quarter $= 2$ quarter.

$$\therefore \quad A = P\left(1+\frac{R}{100}\right)^{n\,(\text{no. of units})}$$

$$= ₹\ 20000\left(1+\frac{4}{100}\right)^2 = ₹\ 20000\left(1+\frac{1}{25}\right)^2$$

$$= ₹\ 20000 \times \frac{26\times 26}{25\times 25} = ₹\ 32 \times 26 \times 26$$

$$= ₹\ 21632$$

Now, C.I. $= A - P$

$= ₹\ (21632 - 20000) = ₹\ 1632$

(c) $P =$ ₹ 28000, $R = 12\%$ p.a. ÷ 4 = 3% per quarter

$n = 9$ months $= 3$ quarters

$$\therefore \quad A = P\left(1+\frac{R}{100}\right)^{n\,(\text{no. of units})}$$

$$= ₹\ 28000\left(1+\frac{3}{100}\right)^3 = ₹\ 28000 \times \frac{103\times 103\times 103}{100\times 100\times 100}$$

$$= ₹\ \frac{28\times 1092727}{1000}$$

$$= ₹\ 30596.36$$

Now, C.I. $= A - P$

$= ₹\ (30596.36 - 28000) = ₹\ 2596.36$

(d) $P = ₹\ 10000,\ R = 10\%$ p.a. $= \frac{10}{4}\% = \frac{5}{2}\%$ per quarter

$n = 6$ months $= 2$ quarters,

$$\therefore\ A = P\left(1+\frac{R}{100}\right)^{n\text{ (no. of units)}}$$

$$= ₹\ 10000\left(1+\frac{5}{2\times 100}\right)^2$$

$$= ₹\ 10000\left(1+\frac{1}{40}\right)^2$$

$$= ₹\ 10000\times\frac{41}{40}\times\frac{41}{40} = ₹\ \frac{168100}{16}$$

$$= ₹\ 10506.25$$

Now, C.I. $= A - P = ₹\ (10506.25 - 10000) = ₹\ 506.25$

4. $P = ₹\ 100000,\ R = 8\%,\ n = 3$ years, $A = ?$

$$\therefore\ A = P\left(1+\frac{R}{100}\right)^n = ₹\ 100000\left(1+\frac{8}{100}\right)^3$$

$$= ₹\ 100000\left(1+\frac{2}{25}\right)^3 = ₹\ 100000\times\frac{27\times 27\times 27}{25\times 25\times 25}$$

$$= ₹\ \frac{32\times 19683}{5} = ₹\ 125971.20$$

Hence, the amount will be paid ₹ 125971.20 back at the end of 3 years.

5. $P = ₹\ 10000,\ n = 2$ years, $R = 5\%,\ A = ?$

$$\therefore\ A = P\left(1+\frac{R}{100}\right)^n = ₹\ 10000\left(1+\frac{5}{100}\right)^2$$

$$= ₹\ 10000\left(1+\frac{1}{20}\right)^2 = ₹\ 10000\times\frac{21}{20}\times\frac{21}{20}$$

$$= ₹25\times 21\times 21 = ₹\ 11025$$

Hence, Rashmi will receive ₹ 11025 after 2 years.

6. $P = ₹\ 5000,\ R = 8\frac{1}{3}\% = \frac{25}{3}\%,\ n = 2$ years $A = ?$

$$\therefore\ A = P\left(1+\frac{R}{100}\right)^n$$

$$= ₹\ 5000\left(1+\frac{25}{3\times 100}\right)^2 = ₹\ 5000\left(1+\frac{1}{12}\right)^2$$

$$= ₹\ 5000\times\frac{13}{12}\times\frac{13}{12} = ₹\ 5000\times\frac{169}{144}$$

$$= ₹\ \frac{845000}{144} = ₹\ 5868.05$$

Hence, Guneet will pay ₹ 5868.05 to Anju after 2 years.

7. $P = ₹\ 15625,\ R = 16\%,\ n = 2\frac{1}{4}$ years, $A = ?$

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n$

In this case, the time is in fractions.

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^q \left(1+\frac{\frac{m}{n}R}{100}\right)$ where, $q = 2$ and $\frac{m}{n} = \frac{1}{4}$

$= ₹\ 15625\left(1+\frac{16}{100}\right)^2 \left(1+\frac{\frac{1}{4}\times 16}{100}\right)$

$= ₹\ 15625\left(1+\frac{4}{25}\right)^2 \left(1+\frac{1}{25}\right) = ₹\ 15625\left(\frac{29}{25}\right)^2\left(\frac{26}{25}\right)$

$= ₹\ 15625 \times \frac{29}{25}\times\frac{29}{25}\times\frac{26}{25} = ₹29\times 29\times 26 = ₹\ 21866$

Hence, Abhishek will pay ₹ 21866 after $2\frac{1}{4}$ years.

8. S.I. $= ₹\ 6750,\ R = 6\frac{2}{3}\% = \frac{20}{3}\%,\ n = 3$ years

$P = ?,\qquad$ C.I. $= ?$

$\because \qquad \text{S.I.} = \frac{P\times R\times T}{100}$

$₹\ 6750 = \frac{P\times\frac{20}{3}\times 3}{100}$

$P = ₹\ 6750 \times 5 = ₹\ 33750$

We know that,

$\text{C.I.} = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\} = ₹\ 33750\left\{\left(1+\frac{20}{3\times 100}\right)^3 - 1\right\}$

$= ₹\ 33750\left\{\left(1+\frac{1}{15}\right)^3 - 1\right\} = ₹\ 33750\left(\frac{16\times 16\times 16}{15\times 15\times 15} - 1\right)$

$= ₹\ 33750\left(\frac{4096}{3375} - 1\right)$

$= ₹\ 33750\left(\frac{4096-3375}{3375}\right)$

$= ₹\ 10\times 721 = ₹\ 7210$

Hence, the compound interest will be ₹ 7210.

9. Since, the interest is compounded half yearly.

Therefore, $R = 12.5\% \div 2$ half yearly $= \frac{25}{4}\%$ per unit

$P = ₹\ 32500$

Time $= 1\frac{1}{2}$ year $= 3$ half year or 3 conversion periods.

$\therefore$ Amount $= P\left(1 + \frac{R}{100}\right)^n = ₹\ 32500\left(1 + \frac{25}{4 \times 100}\right)^3$

$= ₹\ 32500\left(1 + \frac{1}{16}\right)^3$

$= ₹\ 32500 \times \frac{17 \times 17 \times 17}{16 \times 16 \times 16}$

$A = ₹\ 38982.54$

$\therefore$ C.I. $= A - P = ₹(38982.54 - 32500) = ₹\ 6482.54$

Hence, Golu will be paid ₹ 6482.54 as interest to Polu after $1\frac{1}{2}$ years.

10. $P = ₹\ 18000$, $R = 15\% \div 2$ half yearly $= \frac{15}{2}\%$ per unit

$n = 1$ year $= 2$ half year or 2 conversion periods

$\therefore$ Amount $= P\left(1 + \frac{R}{100}\right)^n$

$= ₹\ 18000\left(1 + \frac{15}{2 \times 100}\right)^2$

$= ₹\ 18000\left(1 + \frac{3}{40}\right)^2 = ₹\ 18000 \times \frac{43}{40} \times \frac{43}{40}$

$= ₹\ \frac{45 \times 43 \times 43}{4} = ₹\ 20801.25$

Hence, the amount is ₹ 20801.25

11. $P = ₹\ 5000$, $R = 8\frac{1}{3}\% = \frac{25}{3}\%$, $n = 2$ years, $A = ?$

$\therefore A = P\left(1 + \frac{R}{100}\right)^n$

$= ₹\ 5000\left(1 + \frac{25}{3 \times 100}\right)^2 = ₹\ 5000\left(1 + \frac{1}{12}\right)^2$

$= ₹\ 5000\left(\frac{13}{12}\right)^2$

$= ₹5000 \times \frac{13}{12} \times \frac{13}{12} = ₹\ 5868.06$

Hence, Jyoti will be paid ₹ 5868.06 to Rakhi after 2 years.

12. $P = ₹\ 10000,\ R = 10\%$ p.a., $n = 12$ months $= 1$ year

(a) Since, the interest is compounded annually.

$$\therefore \quad \text{C.I.} = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

$$= ₹\ 10000\left\{\left(1+\frac{10}{100}\right)^1 - 1\right\}$$

$$= ₹\ 10000\left\{\left(\frac{11}{10}\right) - 1\right\}$$

$$= ₹\ 10000\left(\frac{11-10}{10}\right) = ₹\ 1000$$

(b) Since, the interest is compounded half-yearly.

Then, $R = 10\% \div 2 = 5\%$ per unit

$n = 1$ year $= 2$ half year or 2 conversion periods.

$$\therefore \quad \text{C.I.} = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

$$= ₹\ 10000\left\{\left(1+\frac{5}{100}\right)^2 - 1\right\} = ₹\ 10000\left\{\left(1+\frac{1}{20}\right)^2 - 1\right\}$$

$$= ₹\ 10000\left\{\frac{21\times 21}{20\times 20} - 1\right\} = ₹\ 10000\left(\frac{441-400}{400}\right)$$

$$= ₹\ 10000 \times \frac{41}{400} = ₹\ 1025$$

(c) Since, the interest is compounded quarterly

Then, $R = 10\% \div 4 = \frac{5}{2}\%$ per unit

$n = 12$ months $= 4$ quarters

$$\therefore \quad \text{C.I.} = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

$$= ₹\ 10000\left\{\left(1+\frac{5}{2\times 100}\right)^4 - 1\right\}$$

$$= ₹\ 10000\left\{\left(1+\frac{1}{40}\right)^4 - 1\right\} = ₹\ 10000\left\{\left(\frac{41}{40}\right)^4 - 1\right\}$$

$$= ₹\ 10000\left(\frac{2825761-2560000}{2560000}\right)$$

$$= ₹\ \frac{265761}{256}$$

$$= ₹\ 1038.13$$

Exercise 8.7

1. $A = ₹\ 5832,\ n = 2$ years, $R = 8\%,\ P = ?$

$$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n$$

$$₹\ 5832 = P\left(1+\frac{8}{100}\right)^2$$

$$₹\ 5832 = P\left(1+\frac{2}{25}\right)^2$$

$$₹\ 5832 = P\left(\frac{27}{25}\right)^2$$

$$P = ₹\ \frac{5832\times 25\times 25}{27\times 27}$$

$$P = ₹\ 8\times 625 = ₹\ 5000$$

Hence, the certain sum is ₹ 5000.

2. $A = ₹\ 10240,\ n = 2$ years, $R = 6\frac{2}{3}\% = \frac{20}{3}\%,\ P = ?$

$$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n$$

$$₹\ 10240 = P\left(1+\frac{20}{3\times 100}\right)^2$$

$$₹\ 10240 = P\left(1+\frac{1}{15}\right)^2$$

$$₹\ 10240 = P\left(\frac{16}{15}\right)^2$$

$$P = ₹\ \frac{15\times 15\times 10240}{16\times 16}$$

$$= ₹\ 225\times 40 = ₹\ 9000$$

Hence, the sum of money is ₹ 9000.

3. $A = ₹\ 774.40,\ P = ₹\ 640,\ T = 2$ years, $R = ?$

$$\because \quad A = P\left(1+\frac{R}{100}\right)^n$$

$$\therefore \quad ₹\ 774.40 = ₹\ 640\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{774.40}{640} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{7744}{6400} = \left(1 + \frac{R}{100}\right)^2$$

$$\Rightarrow \quad \left(1 + \frac{R}{100}\right) = \sqrt{\frac{7744}{6400}}$$

$$\Rightarrow \quad 1 + \frac{R}{100} = \frac{88}{80}$$

$$\frac{R}{100} = \frac{88}{80} - 1$$

$$\Rightarrow \quad \frac{R}{100} = \frac{88 - 80}{80}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{8}{80}$$

$$\Rightarrow \quad R = \frac{800}{80}\%$$

$$\Rightarrow \quad R = 10\%$$

Hence, the rate of interest is 10% p.a.

4. $P = ₹\ 2000,\ A = ₹\ 2205,\ n = 2$ years, $R = ?$

$$\because \quad A = P\left(1 + \frac{R}{100}\right)^n$$

$$\therefore \quad 2205 = 2000\left(1 + \frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{2205}{2000} = \left(1 + \frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{441}{400} = \left(1 + \frac{R}{100}\right)^2$$

$$\Rightarrow \quad \left(1 + \frac{R}{100}\right) = \sqrt{\frac{441}{400}}$$

$$\Rightarrow \quad 1 + \frac{R}{100} = \frac{21}{20}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{21}{20} - 1$$

$$\Rightarrow \quad \frac{R}{100} = \frac{21 - 20}{20}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{1}{20}$$

$$\Rightarrow \quad R = \frac{100}{20}\%$$

$$\Rightarrow \quad R = 5\%$$

Hence, the rate of interest is 5% p.a.

5. S.I. $=$ ₹ 2100, $R = 14\%$, $T = 2$ years, C.I. $= ?$

$\because$ $$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$₹\ 2100 = \frac{P \times 14 \times 2}{100}$$

$$P = ₹\ \frac{2100 \times 25}{7}$$

$$P = ₹\ 300 \times 25 = ₹\ 7500$$

So, $$\text{C.I.} = P\left\{\left(1 + \frac{R}{100}\right)^n - 1\right\} = ₹\ 7500\left\{\left(1 + \frac{14}{100}\right)^2 - 1\right\}$$

$$= ₹\ 7500\left\{\left(\frac{57}{50}\right)^2 - 1\right\} = ₹\ 7500\left(\frac{3249 - 2500}{2500}\right)$$

$$= ₹\ 7500 \times \frac{749}{2500} = ₹\ 2247$$

Hence, the C.I. is ₹ 2247.

6. $A_1 =$ ₹ 7396, $n_1 = 2$ years, $A_2 =$ ₹ 7950.70, $n_2 = 3$ years, $R = ?$

We know that, $$A = P\left(1 + \frac{R}{100}\right)^n$$

$\therefore$ $$A_1 = P\left(1 + \frac{R}{100}\right)^{n_1} \quad \ldots\text{(i)}$$

$$A_2 = P\left(1 + \frac{R}{100}\right)^{n_2} \quad \ldots\text{(ii)}$$

Divide (ii) by (i), we get

$$\frac{A_2}{A_1} = \frac{\left(1 + \frac{R}{100}\right)^{n_2}}{\left(1 + \frac{R}{100}\right)^{n_1}}$$

$$\frac{7950.70}{7396} = \frac{\left(1 + \frac{R}{100}\right)^3}{\left(1 + \frac{R}{100}\right)^2}$$

$$\frac{7950.70}{7396} = 1 + \frac{R}{100}$$

$$\frac{R}{100} = \frac{7950.70 - 7396}{7396}$$

$$\frac{R}{100} = \frac{554.70}{7396}$$

$$R = \frac{554.70}{7396} \times 100 = \frac{55470}{7396}$$

$$R = \frac{15}{2}\% = 7\frac{1}{2}\%$$

Hence, the rate of interest is $7\frac{1}{2}$% p.a.

7. Given, C.I. − S.I. = ₹ 180, $R = 10\%$, $n = T = 1$ years.

$$\Rightarrow \quad P\left\{\left(1+\frac{5}{100}\right)^2 - 1\right\} - \frac{P \times 10 \times 1}{100} = ₹\ 180$$

[Since, the interest is half yearly.]

$$\Rightarrow \quad P\left\{\left(1+\frac{1}{20}\right)^2 - 1\right\} - \frac{P}{10} = ₹\ 180$$

$$\Rightarrow \quad P\left\{\left(\frac{21}{20}\right)^2 - 1\right\} - \frac{P}{10} = ₹\ 180$$

$$\Rightarrow \quad P\left(\frac{441-400}{400}\right) - \frac{P}{10} = ₹\ 180$$

$$\Rightarrow \quad P\left[\frac{41}{400} - \frac{1}{10}\right] = ₹\ 180$$

$$\Rightarrow \quad P\left[\frac{41-40}{400}\right] = ₹\ 180$$

$$\Rightarrow \quad P = ₹\ \frac{180 \times 400}{(41-40)}$$

$$\Rightarrow \quad P = ₹\ 72000$$

Hence, the sum of money is ₹ 180.

8. $A = ₹\ 784$, $P = ₹\ 625$, $n = 2$ years, $R = ?$

$$\because \quad A = P\left(1+\frac{R}{100}\right)^n$$

$$\therefore \quad 784 = 625\left(1+\frac{R}{100}\right)^2$$

$$\frac{784}{625} = \left(1+\frac{R}{100}\right)^2$$

$$\left(1+\frac{R}{100}\right) = \sqrt{\frac{784}{625}}$$

$$1+\frac{R}{100} = \frac{28}{25}$$

$$R = \left(\frac{28-25}{25}\right) \times 100\%$$

$R = 3 \times 4 = 12\%$

Hence, the rate of interest is 12% p.a.

9. $P = ₹\ 500,\ A = ₹\ 605,\ R = 10\%,\ n = ?$

$\because \qquad A = P\left(1 + \frac{R}{100}\right)^n$

$\therefore \qquad 605 = 500\left(1 + \frac{10}{100}\right)^n$

$\Rightarrow \qquad \frac{605}{500} = \left(1 + \frac{1}{10}\right)^n$

$\Rightarrow \qquad \frac{121}{100} = \left(\frac{11}{10}\right)^n$

$\Rightarrow \qquad \left(\frac{11}{10}\right)^2 = \left(\frac{11}{10}\right)^n$

$\because$ Base $\frac{11}{10}$ is the same on both the sides.

$\therefore \qquad \text{Power} = \text{Power}$

$n = 2.$

Hence, the required time is 2 years.

10. $P = ₹\ 8000,\ A = ₹\ 8820,\ R = 20\%,\ n = ?$

Since, the interest is compounded quarterly.

So, $R = 20\% \div 4 = 5\%$ per unit.

$\because \qquad A = P\left(1 + \frac{R}{100}\right)^{4n}$

$$8820 = 8000\left(1 + \frac{5}{100}\right)^{4n}$$

$$\frac{8820}{8000} = \left(1 + \frac{1}{20}\right)^{4n}$$

$$\left(\frac{441}{400}\right) = \left(\frac{21}{20}\right)^{4n}$$

$\Rightarrow \qquad \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^{4n}$

$\because$ Base is the same on both sides

$\therefore \quad 4n = 2 \text{ years} \qquad \Rightarrow \qquad n = \frac{2}{4} \quad \Rightarrow \quad n = \frac{1}{2}$ years.

Hence, the required time is half year or six months.

11. $P = ₹\ 4400,\ A = ₹\ 4576,\ R = 8\%,\ n = ?$

Since, the interest is compounded half yearly.

So, $\qquad R = 8\% \div 2 = 4\%$

$\because \quad A = P\left(1+\frac{R}{100}\right)^{2n}$

$\therefore \quad 4576 = 4400\left(1+\frac{4}{100}\right)^{2n}$

$\Rightarrow \quad \frac{4576}{4400} = \left(1+\frac{1}{25}\right)^{2n}$

$\Rightarrow \quad \frac{416}{400} = \left(\frac{26}{25}\right)^{2n}$

$\Rightarrow \quad \left(\frac{26}{25}\right)^{1} = \left(\frac{26}{25}\right)^{2n}$

$\because$ Base is the same on both sides

$\therefore \quad 2n = 1$ years $\quad \Rightarrow \quad n = \frac{1}{2}$ years.

Hence, the required time is half year or six months.

12. Let ₹ x be the sum, $n = 2$ years, $R = 10\%$

Given, C.I. − S.I. = ₹ 10

So, according to the question,

$$x\left\{\left(1+\frac{10}{100}\right)^2 - 1\right\} - \frac{x \times 10 \times 2}{100} = ₹\ 10$$

$$x\left\{\left(1+\frac{1}{10}\right)^2 - 1\right\} - \frac{x}{5} = ₹\ 10$$

$$x\left\{\left(\frac{11}{10}\right)^2 - 1\right\} - \frac{x}{5} = ₹\ 10$$

$$x\left(\frac{121-100}{100}\right) - \frac{x}{5} = ₹\ 10$$

$$x \times \frac{21}{100} - \frac{x}{5} = ₹\ 10$$

$$\frac{21x - 20x}{100} = ₹\ 10$$

$$x = ₹\ 10 \times 100$$

$$x = ₹\ 1000$$

Hence, the sum of money is ₹ 1000.

13. Given, C.I. = ₹ 110, $n = T = 2$ years, S.I. = ₹ 100

$$P\left\{\left(1+\frac{R}{100}\right)^2 - 1\right\} = ₹\ 110 \quad \ldots(i)$$

$$\frac{P \times R \times 2}{100} = ₹\ 100 \quad \ldots(ii)$$

Divide (i) by (ii), we get

$$\frac{P\left\{\left(1+\frac{R}{100}\right)^2-1\right\}}{2\frac{PR}{100}}=\frac{110}{100}$$

$$\Rightarrow \quad \frac{\left(1+\frac{R^2}{10000}+\frac{2R}{100}-1\right)}{\frac{2R}{100}}=\frac{11}{10}$$

$$\Rightarrow \quad \frac{\frac{R}{100}\left(\frac{R}{100}+2\right)}{2\frac{R}{100}}=\frac{11}{10}$$

$$\Rightarrow \quad \frac{R}{100}+2=\frac{22}{10}$$

$$\Rightarrow \quad \frac{R}{100}=\frac{22}{10}-2=\frac{22-20}{10}$$

$$R=\frac{2\times 100}{10}\%$$

$$R=20\%$$

Since, $$\text{S.I.}=\frac{P\times R\times T}{100}$$

$$100=\frac{P\times 20\times 2}{100}$$

$$P=₹\ 10000\div 40$$

$$P=₹\ 250$$

Hence, the sum of money is ₹ 250 and rate of interest is 20% p.a.

Exercise 8.8

1. $P_0=80000,\ R=\frac{75}{1000}\times 100=7.5\%,\ n=2$ years, $P_1=?$

$$\therefore \quad P_1=P_0\left(1+\frac{R}{100}\right)^n=80000\left(1+\frac{75}{1000}\right)^2$$

$$P_1=80000\left(1+\frac{3}{40}\right)^2=80000\times\frac{43\times 43}{40\times 40}$$

$$P_1=50\times 1849$$

$$P_1=92450$$

Hence, the population of the town will be 92450 after 2 years.

2. $P_0 = 12000, R = 10\% \; n = 3$ years, $P_1 = ?$

$$\therefore \quad P_1 = P_0\left(1+\frac{R}{100}\right)^n = 12000\left(1+\frac{10}{100}\right)^3 = 12000\left(1+\frac{1}{10}\right)^3$$

$$P_1 = 12000 \times \frac{11\times 11\times 11}{10\times 10\times 10}$$

$$P_1 = 12\times 1331 \quad \Rightarrow \quad P_1 = 15972$$

Hence, the population of the town will be 15972 after 3 years.

3. $P_0 =$ ₹ 10000, $R = 10\%$, $n = 2$ years, $P_1 = ?$

$\because$ Its value depreciates annually.

$$\therefore \quad P_1 = P_0\left(1-\frac{r}{100}\right)^n$$

$$P_1 = 10000\left(1-\frac{10}{100}\right)^2$$

$$P_2 = ₹\ 10000\left(1-\frac{1}{10}\right)^2 = ₹\ 10000 \times \frac{9\times 9}{10\times 10}$$

$$= ₹\ 100 \times 81 = ₹\ 8100$$

Hence, the value of the machine will be ₹ 8100 after 2 years.

4. $P_0 =$ ₹ 64000, $R = 2.5\%$, $n = 3$ years., $P_1 = ?$

$\because$ Its value deprecated annually.

$$\therefore \quad P_1 = P_o\left(1-\frac{R}{100}\right)^n = ₹\ 64000\left(1-\frac{2.5}{100}\right)^3$$

$$= ₹\ 64000\left(1-\frac{1}{40}\right)^3 = ₹\ 64000\left(\frac{40-1}{40}\right)^3$$

$$= ₹\ 64000 \times \frac{39\times 39\times 39}{40\times 40\times 40}$$

$$= ₹\ 39 \times 39 \times 39 = ₹\ 59319$$

Hence, the value of the T.V. will be ₹ 59319 after 3 years.

And the value of the T.V. gone down = ₹ (64000 − 59319) = ₹ 4681.

5. $P_0 = 1000$

Since, it is increases at the rate of 5% every six months.

So, R= 5%, $n = 18$ months = 3 half year $P_t = ?$

$$\therefore \quad P_t = P_o\left(1+\frac{R}{100}\right)^3 = 1000\left(1+\frac{5}{100}\right)^3$$

$$= 1000\left(1+\frac{1}{20}\right)^3 = 1000\left(\frac{21}{20}\right)^3$$

$$= 1000\times\frac{21\times 21\times 21}{20\times 20\times 20} = 1157.6 \text{ or } 1158$$

Hence, the population of I block will be 1158 after 18 months.

Multiple Choice Questions

Tick (✓) the correct option :

1. (a), **2.** (b), **3.** (c), **4.** (b), **5.** (a), **6.** (a),
7. (a), **8.** (b), **9.** (b), **10.** (d), **11.** (d), **12.** (b).

Brain Teaser

1. Find :

(i) Express 65% as a fraction.

$$65\% = \frac{65}{100} = \frac{13}{20}$$

(ii) Given : 5% of y is 4.

$$\therefore \quad \frac{5}{100} \times y = 4$$

$$y = \frac{4 \times 100}{5}$$

$$y = 80$$

Hence, the value of y is 80.

2. Write 'T' for True or 'F' for False :

(i) T, (ii) F, (iii) T;

9 Algebraic Expressions and Indentities

Exercise 9.1

1. Classify the following algebraic expressions as monomials, binomials, and trinomials.

(a) $4x - y^2$ (Binomials) (b) $p^2 + q^2$ (Binomials)

(c) y^3 (Monomials) (d) $x^2 - 2xy + 24$ (Trinomials)

2. (a) $3x^2, -7x^2, \frac{1}{5}x^2$

$$= 3x^2 + (-7x^2) + \frac{1}{5}x^2 = \frac{3x^2}{1} - \frac{7x^2}{1} + \frac{x^2}{5}$$

$$= \frac{15x^2 - 35x^2 + x^2}{5} = \frac{16x^2 - 35x^2}{5} = \frac{-19}{5}x^2$$

(b) $-19p^2 + 3q^2 - 5r^2, 2q^2 - 5p^2 + 17r^2, \frac{-1}{7}p^2 + \frac{3}{5}q^2 - r^2$

$$= -19p^2 + 3q^2 - 5r^2 + 2q^2 - 5p^2 + 17r^2 - \frac{1}{7}p^2 + \frac{3}{5}q^2 - r^2$$

$$= \frac{-19p^2}{1} - \frac{5p^2}{1} - \frac{1p^2}{7} + \frac{3q^2}{1} + \frac{2q^2}{1} + \frac{3q^2}{5} - 5r^2 + 17r^2 - r^2$$

$$=\left(\frac{-133p^2-35p^2-1p^2}{7}\right)+\left(\frac{15q^2+10q^2+3q^2}{5}\right)+11r^2$$

$$=\frac{-169}{7}p^2+\frac{28}{5}q^2+11r^2$$

(c) $\frac{1}{3}x^2-\frac{2}{3}y+11z,\ \frac{-7}{9}x^2-13z+y,\ \frac{23}{3}x^2-4y$

$$=\frac{1}{3}x^2-\frac{2}{3}y+11z-\frac{7}{9}x^2-13z+y+\frac{23}{3}x^2-4y$$

$$=\frac{1}{3}x^2-\frac{7}{9}x^2+\frac{23}{3}x^2-\frac{2}{3}y+\frac{y}{1}-\frac{4y}{1}+11z-13z$$

$$=\frac{3x^2-7x^2+69x^2}{9}+\left(\frac{-2y+3y-12y}{3}\right)-2z$$

$$=\frac{65x^2}{9}-\frac{11y}{3}-2z$$

3. (a) $-6y^2x^2$ from $-19y^2x^2$

$$=-19y^2x^2-(-6y^2x^2)$$

$$=-19y^2x^2+6y^2x^2=-13y^2x^2$$

(b) $\frac{-1}{5}xy^2z$ from $17xy^2z$

$$=17xy^2z-\left(-\frac{1}{5}xy^2z\right)=\frac{17xy^2z}{1}+\frac{1}{5}xy^2z$$

$$=\frac{85xy^2z+xy^2z}{5}=\frac{86}{5}xy^2z$$

(c) $p^2-2p+\frac{1}{3}$ from $-p^2+2p-\frac{1}{3}$

$$=\left(-p^2+2p-\frac{1}{3}\right)-\left(p^2-2p+\frac{1}{3}\right)$$

$$=-p^2+2p-\frac{1}{3}-p^2+2p-\frac{1}{3}=-2p^2+4p-\frac{2}{3}$$

(d) -1 from $3x^2+7y^2-8z^2$

$$=3x^2+7y^2-8z^2-(-1)=3x^2+7y^2-8z^2+1$$

(e) $3a+7b-6c+4$ from $-3a-7b+6c-4$

$$=(-3a-7b+6c-4)-(3a+7b-6c+4)$$

$$=-3a-7b+6c-4-3a-7b+6c-4$$

$$=-6a-14b+12c-8$$

4. Let length of rectangle $=(2x+y)$ unit

And, Breadth of rectangle $=(3y-2x)$ unit

$\therefore$ Perimeter of rectangle $=2(l+b)$

$$=2(2x+y+3y-2x)$$

$$=2\times 4y$$

$$=8y \text{ unit.}$$

Exercise 9.2

1. (a) $(6a^2b)(-2b^2c)(3ac^2)$

$= 6a^2b \times (-2b^2c) \times (3ac^2)$

$= -36a^3b^3c^3$

(b) $(ab)(bc)(ca)$

$= (ab) \times (bc) \times (ca)$

$= a^2b^2c^2$

(c) $(-2xy^2)(5y)(-3z^2)$

$= (-2xy^2) \times 5y \times (-3z^2)$

$= +30xy^3z^2$

(d) $\left(\frac{5}{9}ab\right)\left(\frac{9}{7}bc\right)\left(\frac{-7}{5}ca\right)$

$= \frac{5}{9}ab \times \frac{9}{7}bc \times \left(\frac{-7}{5}ca\right)$

$= -a^2b^2c^2$

2. $= (-8x^2y^3) \times \left(\frac{1}{5}xy^2\right)$

$= \frac{-8}{5}x^3y^5$

Now, putting, $x = -1$ and $y = 2$, we have

$= \frac{-8}{5} \times (-1)^3 \times (2)^5$

$= \frac{-8}{5} \times -1 \times -1 \times -1 \times 2 \times 2 \times 2 \times 2 \times 2$

$= \frac{8 \times 32}{5} = \frac{256}{5}$

3. We have,

$= (3p^2q) \times (8q^3)$

$= 24p^2q^4$

Now, putting $p = 1$ and $q = \frac{-1}{4}$, we have

$= 24 \times (1)^2 \times \left(\frac{-1}{4}\right)^4$

$= 24 \times 1 \times 1 \times \frac{-1}{4} \times \frac{-1}{4} \times \frac{-1}{4} \times \frac{-1}{4}$

$= \frac{24}{256} = \frac{3}{32}$

4. (a) $(-3x)(2x^2 + 6x - 7)$

$= (-3x) \times (2x^2) + (-3x) \times (6x) + (-3x) \times (-7)$

$= -6x^3 - 18x^2 + 21x$

(b) $\frac{1}{2}xy(x^2 - 2xy + y^2)$

$= \frac{1}{2}xy \times x^2 + \frac{1}{2}xy \times (-2xy) + \frac{1}{2}xy \times y^2$

$= \frac{1}{2}x^3y - x^2y^2 + \frac{1}{2}xy^3$

(c) $a^2(a^3+3a^2b+b^3+3ab^2)$

$= a^2\times a^3 + a^2\times 3a^2b + a^2\times b^3 + a^2\times 3ab^2$

$= a^5 + 3a^4b + a^2b^3 + 3a^3b^2$

5. $a^2b^2c^2 = (ab)\times(bc)\times(ca)$

Putting $a=3$, and $b=4$ on both sides.

$3^2\times 4^2\times c^2 = (3\times 4)\times(4\times c)\times(c\times 3)$

$9\times 16\times c^2 = 12\times 4c\times 3c$

$144c^2 = 144c^2$

L.H.S. = R.H.S. **Proved**

6. (a) $2x(3x+y^2) = 2x\times 3x + 2x\times y^2$

$= 6x^2 + 2xy^2$

(b) $(-3y)(x^2+3xy) = (-3y)\times(x^2) + (-3y)\times(3xy)$

$= -3x^2y - 9xy^2$

(c) $3a^2(4a-5a^2) = 3a^2\times 4a + 3a^2\times(-5a^2)$

$= 12a^3 - 15a^4$

(d) $-8a^2b(-3a^2-2b) = (-8a^2b)\times(-3a^2) + (-8a^2b)\times(-2b)$

$= 24a^4b + 16a^2b^2$

(e) $\frac{-5}{9}abc\left(\frac{18}{15}a^2bc - \frac{3}{10}abc^2\right)$

$= \frac{-5}{9}abc\times\frac{18}{15}a^2bc + \left(\frac{-5}{9}abc\right)\times\left(\frac{-3}{10}abc^2\right)$

$= \frac{-2}{3}a^3b^2c^2 + \frac{1}{6}a^2b^2c^3$

(f) $7a(0.1a^2 - 0.5b) = 7a\times 0.1\ a^2 - 7a\times 0.5b$

$= 0.7a^3 - 3.5ab$

Exercise 9.3

1. (a) $(x^2-a^2)(x-a) = x^2(x-a) - a^2(x-a)$

$= x^3 - ax^2 - a^2x + a^3$

(b) $\left(x^3+\frac{1}{x^3}\right)\left(x+\frac{1}{2}\right) = x^3\left(x+\frac{1}{2}\right) + \frac{1}{x^3}\left(x+\frac{1}{2}\right)$

$= x^4 + \frac{x^3}{2} + \frac{1}{x^2} + \frac{1}{2x^3}$

(c) $(a^2b+ab^2)(b^2c+c^2b) = a^2b(b^2c+c^2b) + ab^2(b^2c+c^2b)$

$= a^2b^3c + a^2b^2c^2 + ab^4c + ab^3c^2$

(d) $(2x-y)(3x+y^2) = 2x(3x+y^2) - y(3x+y^2)$

$= 6x^2 + 2xy^2 - 3xy - y^3$

$= 6x^2 - 3xy + 2xy^2 - y^3$

(e) $(x-3y)(x^2+3xy) = x(x^2+3xy) - 3y(x^2+3xy)$

$= x^3 - 9xy^2$

(f) $\left(\frac{2}{7}x+\frac{3}{5}y\right)(x^2+y^2) = \frac{2}{7}x(x^2+y^2) + \frac{3}{5}y(x^2+y^2)$

$$= \frac{2}{7}x^3 + \frac{2}{7}xy^2 + \frac{3}{5}x^2y + \frac{3}{5}y^3$$
$$= \frac{2}{7}x^3 + \frac{3}{5}x^2y + \frac{2}{7}xy^2 + \frac{3}{5}y^3$$

2. (a) $(3x^2 + 2y^2)(x + y) = 3x^2(x + y) + 2y^2(x + y)$
$$= 3x^3 + 3x^2y + 2xy^2 + 2y^3$$

Putting $x = -1$ and $y = -2$, we have

$(3x^2 + 2y^2)(x + y) = [3\times(-1)^2 + 2\times(-2)^2][(-1) + (-2)]$
$$= [3\times 1 + 2\times 4][-1 - 2]$$
$$= (3 + 8)(-3)$$
$$= 11\times(-3) \Rightarrow -33$$

And, $3x^3 + 3x^2y + 2xy^2 + 2y^3$
$$= 3\times(-1)^3 + 3\times(-1)^2\times(-2) + 2\times(-2)\times(-2)^2 + 2\times(-2)^3$$
$$= 3\times(-1) + 3\times 1\times(-2) + 2\times(-1)\times 4 + 2\times(-8)$$
$$= -3 - 6 - 8 - 16 \Rightarrow -33$$

Hence, L.H.S. = R.H.S. **Verified**.

(b) $(x^2 - y^2)(x^2 + y^2) = x^2(x^2 + y^2) - y^2(x^2 + y^2)$
$$= x^4 + x^2y^2 - x^2y^2 - y^4$$
$$= x^4 - y^4$$

Putting $x = -1$ and $y = -2$, we have

$(x^2 - y^2)(x^2 + y^2) = [(-1)^2 - (-2)^2][(-1)^2 + (-2)^2]$
$$= [1 - 4][1 + 4]$$
$$= (-3)\times(5) \Rightarrow -15$$

And, $x^4 - y^4 = (-1)^4 - (-2)^4$
$$= 1 - 16 \Rightarrow -15$$

Hence, L.H.S. = R.H.S. **Verified**.

(c) $\left(3x^2 + \frac{1}{3}y^2\right)(2y - 3x^2) = 3x^2(2y - 3x^2) + \frac{1}{3}y^2(2y - 3x^2)$
$$= 6x^2y - 9x^4 + \frac{2}{3}y^3 - x^2y^2$$

Putting $x = -1$ and $y = -2$, we have

$\left(3x^2 + \frac{1}{3}y^2\right)(2y - 3x^2)$
$$= \left[3\times(-1)^2 + \frac{1}{3}\times(-2)^2\right][2\times(-2) - 3\times(-1)]$$
$$= \left[3\times 1 + \frac{1}{3}\times 4\right][-4 - 3\times 1] = \left[3 + \frac{4}{3}\right][-4 - 3]$$
$$= \left[\frac{9+4}{3}\right]\times[-7]$$
$$= \frac{13}{3}\times(-7) \Rightarrow \frac{-91}{3}$$

And, $6x^2y - 9x^4 + \frac{2}{3}y^2 - x^2y^2$

$= 6\times(-1)^2\times(-2) - 9\times(-1)^4 + \frac{2}{3}\times(-2)^3 - (-1)^2\times(-2)^2$

$= 6\times 1\times(-2) - 9\times 1 + \frac{2}{3}\times(-8) - 1\times 4$

$= -12 - 9 - \frac{16}{3} - 4 = -25 - \frac{16}{3}$

$= \frac{-75-16}{3} \Rightarrow \frac{-91}{3}$

Hence, L.H.S. = R.H.S. **Verified**.

(d) $(x^4 - y^4)(x+y) = x^4(x+y) - y^4(x+y) = x^5 + x^4y - xy^4 - y^5$

Putting $x = -1$ and $y = -2$ we have

$(x^4 - y^4)(x+y) = [(-1)^4 - (-2)^4][(-1)+(-2)]$

$= [1-16][-1-2]$

$= [-15][-3] \Rightarrow 45$

And, $x^5 + x^4y - xy^4 - 5^5$

$= (-1)^5 + (-1)^4\times(-2) - (-1)\times(-2)^4 - (-2)^5$

$= -1 + 1\times(-1) + 1\times 16 - (-32)$

$= -1 - 2 + 16 + 32$

$= -3 + 48 \Rightarrow 45$

Hence, L.H.S. = R.H.S. **Verified**.

(e) $\left(\frac{1}{2}x - y\right)\left(\frac{3}{5}x + y\right) = \frac{1}{2}x\left(\frac{3}{5}x + y\right) - y\left(\frac{3}{5}x + y\right)$

$= \frac{3}{10}x^2 + \frac{1}{2}xy - \frac{3}{5}xy - y^2$

$= \frac{3}{10}x^2 + \left(\frac{5-6}{10}\right)xy - y^2 = \frac{3}{10}x^2 - \frac{1}{10}xy - y^2$

Putting $x = -1$ and $y = -2$, we have

$\left(\frac{1}{2}x - y\right)\left(\frac{3}{5}x + y\right) = \left[\frac{1}{2}\times(-1) - (-2)\right]\left[\frac{3}{5}(-1) + (-2)\right]$

$= \left[-\frac{1}{2} + 2\right]\left[\frac{-3}{5} - 2\right]$

$= \left[\frac{-1+4}{2}\right]\left[\frac{-3}{5} - 2\right]$

$= \left[\frac{3}{2}\right]\times\left[\frac{-13}{5}\right] \Rightarrow \frac{-39}{10}$

And, $\frac{3}{10}x^2 - \frac{1}{10}xy - y^2 = \frac{3}{10}\times(-1)^2 - \frac{1}{10}\times(-1)\times(-2) - (-2)^2$

$= \frac{3}{10}\times 1 - \frac{1}{10}\times 2 - 4$

$= \frac{3}{10} - \frac{2}{10} - 4$

$= \frac{3-2-40}{10} \Rightarrow \frac{-39}{10}$

Hence, L.H.S.= R.H.S. **Verified**

(f) $(0.7x - 0.6y)(2.3x - 2y) = 0.7x(2.3x - 2y) - 0.6y(2.3x - 2y)$

$= 1.61x^2 - 1.4xy - 1.38xy + 1.2y^2$

$= 1.61x^2 - 2.78xy + 1.2y^2$

Putting $x = -1$ and $y = -2$, we have

$(0.7x - 0.6y)(2.3x - 2y)$

$= [0.7 \times (-1) - 0.6 \times (-2)][2.3 \times (-1) - 2 \times (-2)]$

$= [-0.7 + 1.2][-2.3 + 4]$

$= [0.5] \times [1.7] \Rightarrow 0.85$

And, $1.61x^2 - 2.78xy + 1.2y^2$

$= 1.61 \times (-1)^2 - 2.78 \times (-1) \times (-2) + 1.2 \times (-2)^2$

$= 1.61 \times 1 - 2.78 \times 2 + 1.2 \times 4$

$= 1.61 - 5.56 + 4.8$

$= 6.41 - 5.56 \Rightarrow 0.85$

Hence, L.H.S. =R.H.S. **Verified**.

3. (a) $(3x - 2)(5x^2 + 6x + 2)$

$= 3x(5x^2 + 6x + 2) - 2(5x^2 + 6x + 2)$

$= 15x^3 + 18x^2 + 6x - 10x^2 - 12x - 4$

$= 15x^3 + 8x^2 - 6x - 4$

(b) $(x^2 + y^2 + z^2)(xy + yz)$

$= x^2(xy + yz) + y^2(xy + yz) + z^2(xy + yz)$

$= x^3y + x^2yz + xy^3 + y^3z + xyz^2 + yz^3$

(c) $(x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$

(d) $(5x^2 + y)(3x + 2y)$

$= 5x^2(3x + 2y) + y(3x + 2y)$

$= 15x^3 + 10x^2y + 3xy + 2y^2$

(e) $(x^3 + y^3)(x^2 - xy + y^2)$

$= x^3(x^2 - xy + y^2) + y^3(x^2 - xy + y^2)$

$= x^5 - x^4y + x^3y^2 + x^2y^3 - xy^4 + y^5$

(f) $\left(\frac{3}{5}x^2 - 3y + 2\right)\left(\frac{1}{3}x - y\right)$

$= \frac{3}{5}x^2\left(\frac{1}{3}x - y\right) - 3y\left(\frac{1}{3}x - y\right) + 2\left(\frac{1}{3}x - y\right)$

$= \frac{1}{5}x^3 - \frac{3}{5}x^2y - xy + 3y^2 + \frac{2x}{3} - 2y$

4. (a) $(2x^2+3y+3)(2y-3x^2)$

$=2x^2(2y-3x^2)+3y(2y-3x^2)+3(2y-3x^2)$

$=4x^2y-6x^4+6y^2-9x^2y+6y-9x^2$

$=-6x^4-9x^2-5x^2y+6y+6y^2$

Now, putting $(x=-2)$ and $(y=-5)$

$(2x^2+3y+3)(2y-3x^2)=-6x^4-9x^2-5x^2y+6y+6y^2$

$[2\times(-2)^2+3\times(-5)+3]\times[2\times(-5)-3\times(-2)^2]$

$=-6\times(-2)^4-9\times(-2)^2-5\times(-2)^2\times(-5)+6\times(-5)+6\times(-5)^2$

$[2\times4-15+3]\times[-10-3\times4]$

$=-6\times16-9\times4-5\times4\times(-5)-30+6\times25\,[8-12]\times[-10-2]$

$=-96-36+100-30+150\,(-4)\times(-22)$

$=-132+220+88=88$

L.H.S. = R.H.S. **Verified.**

(b) $(x-y)(x^2+xy+y^2)$

$=x(x^2+xy+y^2)-y(x^2+xy+y^2)$

$=x^3+x^2y+xy^2-x^2y-xy^2-y^3$

$=x^3-y^3$

Now, putting $(x=-2)$ and $(y=-5)$

$(x-y)(x^2+xy+y^2)=x^3-y^3$

$[(-2)-(-5)][(-2)^2+(-2)\times(-5)+(-5)^2]=(-2)^3-(-5)^3$

$[-2+5][4+10+25]=-8-(-125)$

$(3)\times39=-8+125$

$117=117$

L.H.S. = R.H.S. **Verified**

(c) $(2x+3y)(4x^2-6xy+9y^2)$

$=2x(4x^2-6xy+9y^2)+3y(4x^2-6xy+9y^2)$

$=8x^3-12x^2y+18xy^2+12x^2y-18xy^2+27y^3$

$=8x^3+27y^3$

Now, putting $(x=-2)$ and $(y=-5)$

$(2x+3y)(4x^2-6xy+9y^2)=8x^3+27y^3$

$[2\times(-2)+3\times(-5)\times[4\times(-2)^2-6\times(-2)\times(-5)+9\times(-5)^2]$

$=8\times(-2)^3+27\times(-5)^3$

$[-4-15]\times[4\times4-6\times10+9\times25]=8\times(-8)+27\times(-125)$

$[-19]\times[16-16+225]=-64-3375$

$(-19)\times181=-3439$

$-3439=-3439$

L.H.S. = R.H.S. **Verified**

(d) $(x^2+y)(x^4-x^2y+y^2)$

$=x^2(x^4-x^2y+y^2)+y(x^4-x^2y+y^2)$

$=x^6-x^4y+x^2y^2+x^4y-x^2y^2+y^3$

x^6+y^3

Now, putting $(x=-2)$ and $(y=-5)$

$(x^2+y)(x^4-x^2y+y^2)=x^6+y^3$

$$[(-2)^2+(-5)]\times[(-2)^4-(-2)^2\times(-5)+(-5)^2]=(-2)^6+(-5)^3$$
$$[4-5]\times[16+20+25]=64-125$$
$$[-1]\times[61]=-61$$
$$-61=-61$$
L.H.S. = R.H.S. **Verified**

(e) $(3x+y)\left[\frac{4}{3}x^2+\frac{5}{6}xy+\frac{2}{5}y^2\right]$

$$=3x\left[\frac{4}{3}x^2+\frac{5}{6}xy+\frac{2}{5}y^2\right]+y\left[\frac{4}{3}x^2+\frac{5}{6}xy+\frac{2}{5}y^2\right]$$
$$=4x^3+\frac{5}{2}x^2y+\frac{6}{5}xy^2+\frac{4}{3}x^2y+\frac{5}{6}xy^2+\frac{2}{5}y^3$$
$$=4x^3+\left[\frac{5}{2}+\frac{4}{3}\right]x^2y+\left[\frac{6}{5}+\frac{5}{6}\right]xy^2+\frac{2}{5}y^3$$
$$=4x^3+\frac{23}{6}x^2y+\frac{61}{30}xy^2+\frac{2}{5}y^3$$

Now, putting $(x=-2)$ and $(y=-5)$

$$(3x+y)\left(\frac{4}{3}x^2+\frac{5}{6}xy+\frac{2}{5}y^2\right)$$
$$=4x^3+\frac{23}{6}x^2y+\frac{61}{30}xy^2+\frac{2}{5}y^3$$
$$[3\times(-2)+(-5)]\left[\frac{4}{3}\times(-2)^2+\frac{5}{6}\times(-2)\times(-5)+\frac{2}{5}(-5)^2\right]$$
$$=4\times(-2)^3+\frac{23}{6}\times(-2)^2\times(-5)+\frac{61}{30}\times(-2)\times(-5)^2+\frac{2}{5}(-5)^3$$
$$[-6-5]\times\left[\frac{4}{3}\times4+\frac{5}{6}\times10+\frac{2}{5}\times25\right]$$
$$=4\times(-8)+\frac{23}{6}\times(-20)+\frac{61}{30}\times(-50)+\frac{2}{5}\times(-125)$$
$$(-11)\times\left[\frac{16}{3}+\frac{25}{3}+10\right]=-32-\frac{230}{3}-\frac{305}{3}-50$$
$$(-11)\times\left[\frac{16+25+30}{3}\right]=-82-\left[\frac{535}{3}\right]$$
$$(-11)\times\frac{71}{3}=\frac{-246-535}{3}$$
$$\frac{-781}{3}=\frac{-781}{3}$$
L.H.S. = R.H.S. **Verified**

(f) $(x^4-y^4)(x^2-xy+y^2)$

$$=x^4(x^2-xy+y^2)-y^4(x^2-xy+y^2)$$
$$=x^6-x^5y+x^4y^2-x^2y^4+xy^5-y^6$$

Now, putting $(x=-2)$ and $(y=-5)$

$(x^4-y^4)(x^2-xy+y^2)=x^6-x^5y+x^4y^2-x^2y^4+xy^5-y^6$

$[(-2)^4-(-5)^4]\times[(-2)^2-(-2)\times(-5)+(-5)^2]$

$=(-2)^6-(-2)^5\times(-5)+(-2)^4\times(-5)^2-(-2)^2\times(-5)^4$
$+(-2)\times(-5)^5-(-5)^6$

$[16-625]\times[4-10+25]$

$=64-(-32)\times(-5)+16\times25-4\times625+(-2)\times(-3125)-15625$

$(-609)\times(19)=64-160+400-2500+6250-15625$

$-11571=6714-18285$

$-11571=-11571$

L.H.S. = R.H.S. **Verified**

5. (a) $3x^2(3x+2)(4x-1)=(9x^3+6x^2)(4x-1)$
$=9x^3(4x-1)+6x^2(4x-1)$
$=36x^4-9x^3+24x^3-6x^2$
$=36x^4+15x^3-6x^2$

(b) $(2x^2+7)(2-x)(5+x)=(2x^2+7)[2(5+x)-x(5+x)]$
$=(2x^2+7)[10+2x-5x-x^2]$
$=(2x^2+7)(10-3x-x^2)$
$=2x^2(10-3x-x^2)+7(10-3x-x^2)$
$=20x^2-6x^3-2x^4+70-21x-7x^2$
$=-2x^4-6x^3+13x^2-21x+70$

(c) $(3-5x)(8-3x)(2x+5)=[3(8-3x)-5x(8-3x)](2x+5)$
$=(24-9x-40x+15x^2)(2x+5)$
$=(24-49x+15x^2)(2x+5)$
$=24(2x+5)-49x(2x+5)+15x^2(2x+5)$
$=48x+120-98x^2-245x+30x^3+75x^2$
$=30x^3-23x^2-197x+120$

(d) $(x+3)(x-3)(x-1)=[x(x-3)+3(x-3)](x-1)$
$=(x^2-3x+3x-9)(x-1)$
$=(x^2-9)(x-1)$
$=x^2(x-1)-9(x-1)$
$=x^3-x^2-9x+9$

6. (a) $(3y+2)(y-2)-(7y+3)(y-4)$
$=[3y(y-2)+2(y-2)]-[7y(y-4)+3(y-4)]$
$=[3y^2-6y+2y-4]-[7y^2-28y+3y-12]$
$=(3y^2-4y-4)-(7y^2-25y-12)$
$=3y^2-4y-4-7y^2+25y+12=-4y^2+21y+8$

(b) $(2x-3y)(x+y)-(5x+2y)(x-y)$
$=[2x(x+y)-3y(x+y)]-[5x(x-y)+2y(x-y)]$
$=[2x^2+2xy-3xy-3y^2]-[5x^2-5xy+2xy-2y^2]$
$=[2x^2-xy-3y^2]-[5x^2-3xy-2y^2]$
$=2x^2-xy-3y^2-5x^2+3xy+2y^2$
$=-3x^2+2xy-y^2$

(c) $x^2 + (3x - y)(3x + y + y^2)$

$= x^2 + 3x(3x + y + y^2) - y(3x + y + y^2)$

$= x^2 + 9x^2 + 3xy + 3xy^2 - 3xy - y^2 - y^3 = 10x^2 + 3xy^2 - y^2 - y^3$

(d) $(a^2 - 3a + 5)(2a - 3) - (5a^2 + 3a - 3)(a - 1)$

$= [a^2(2a-3) - 3a(2a-3) + 5(2a-3)] - [5a^2(a-1)$
$+3a(a-1) - 3(a-1)]$

$= [2a^3 - 3a^2 - 6a^2 + 9a + 10a - 15] - [5a^3 - 5a^2 + 3a^2 - 3a - 3a + 3]$

$= [2a^3 - 9a^2 + 19a - 15] - [5a^3 - 2a^2 - 6a + 3]$

$= 2a^3 - 9a^2 + 19a - 15 - 5a^3 + 2a^2 + 6a - 3$

$= -3a^3 - 7a^2 + 25a - 18$

Exercise 9.4

1. Divide :

(a) $64x^3y^3z^3$ by $16xyz = \dfrac{64x^3y^3z^3}{16xyz} = 4x^2y^2z^2$

(b) $125x^3y^2z$ by $25xy^2z = \dfrac{125x^3y^2z}{25xy^2z} = 5x^2$

(c) $64xy^3z^2$ by $64x^4yz = \dfrac{64xy^3z^2}{64x^4yz} = \dfrac{y^2z}{x^3}$

(d) $-54(x^2 + y^2 + z^2)$ by $(x^2 + y^2 + z^2) = \dfrac{-54(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)} = -54$

2. Divide :

(a) $a^5 + a^3 + 8a^2$ by a

$= \dfrac{a^5 + a^3 + 8a^2}{a} = \dfrac{a^5}{a} + \dfrac{a^3}{a} + \dfrac{8a^2}{a} = a^4 + a^2 + 8a$

(b) $2x^3 + 5x^2$ by x

$= \dfrac{2x^3 + 5x^2}{x} = \dfrac{2x^3}{x} + \dfrac{5x^2}{x} = 2x^2 + 5x$

(c) $12a^3 + 3a^2 + 4a$ by $4a$

$= \dfrac{12a^3 + 3a^2 + 4a}{4a} = \dfrac{12a^3}{4a} + \dfrac{3a^2}{4a} + \dfrac{4a}{4a} = 3a^2 + \dfrac{3a}{4} + 1$

(d) $x^3y^2 + 8x^2y + 6xy$ by $2xy$

$= \dfrac{x^3y^2 + 8x^2y + 6xy}{2xy} = \dfrac{x^3y^2}{2xy} + \dfrac{8x^2y}{2xy} + \dfrac{6xy}{2xy} = \dfrac{x^2y}{2} + 4x + 3$

(e) $p^4 - p^3 + 5p^2$ by p^2

$= \dfrac{p^4 - p^3 + 5p^2}{p^2} = \dfrac{p^4}{p^2} - \dfrac{p^3}{p^2} + \dfrac{5p^2}{p^2} = p^2 - p + 5$

(f) $6x^3y^2 - 4x^2y + 2xy$ by $\dfrac{2xy}{3}$

$= \dfrac{3(6x^3y^2 - 4x^2y + 2xy)}{2xy} = \dfrac{18x^3y^2}{2xy} - \dfrac{12x^2y}{2xy} + \dfrac{6xy}{2xy} = 9x^2y - 6x + 3$

3. Divide :

(a) $(28x^3 - 9x^2 - 11x + 4)$ by $(7x - 4)$

$$7x-4\overline{)28x^3-9x^2-11x+4}(4x^2+x-1$$
$$28x^3 - 16x^2$$
$$- \quad +$$
$$+7x^2 - 11x$$
$$+7x^2 - 4x$$
$$- \quad +$$
$$-7x + 4$$
$$-7x + 4$$
$$+ \quad -$$
$$\times$$

Hence, $(28x^3 - 9x^2 - 11x + 4) \div (7x - 4) = 4x^2 + x + 1$

(b) $(x^3 + 6x^2 + 12x + 8)$ by $(x + 2)$

$$x+2\overline{)x^3+6x^2+12x+8}(x^2+4x+4$$
$$x^3 + 2x^2$$
$$- \quad -$$
$$4x^2 + 12x$$
$$4x^2 + 8x$$
$$- \quad -$$
$$4x + 8$$
$$4x + 8$$
$$- \quad -$$
$$\times$$

Hence, $(x^3 + 6x^2 + 12x + 8) \div (x + 2) = x^2 + 4x + 4$

(c) $\left(3x^4 - x^3 + 12x^2 - \frac{4}{3}\right)$ by $(3x - 1)$

$$3x-1\overline{)3x^4-x^3+12x^2-\frac{4}{3}}\left(x^3+4x+\frac{4}{3}\right.$$
$$3x^4 - x^3$$
$$- \quad +$$
$$12x^2 - \frac{4}{3}$$
$$12x^2 - 4x$$
$$- \quad +$$
$$4x - \frac{4}{3}$$
$$4x - \frac{4}{3}$$
$$- \quad +$$
$$\times$$

Hence, $(3x^4 - x^3 + 12x^2 - \frac{4}{3}) \div (3x - 1) = x^3 + 4x + \frac{4}{3}$

(d) $a^4 + a^3 - a - 1$ by $(a^2 + a + 1)$

$$
\begin{array}{r|l}
a^2 + a + 1 & a^4 + a^3 - a - 1 \;(\; a^2 - 1 \\
 & \underline{\overset{-}{a^4} \overset{-}{+ a^3} \overset{-}{+ a^2}} \\
 & -a^2 - a - 1 \\
 & \underline{\overset{+}{-a^2} \overset{+}{- a} \overset{+}{- 1}} \\
 & \times
\end{array}
$$

Hence, $(a^4 + a^3 - a - 1) \div (a^2 + a + 1) = a^2 - 1$

(e) $x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

$$
\begin{array}{r|l}
x^3 + 1 & x^5 + x^4 + x^3 + x^2 + x + 1 \;(\; x^2 + x + 1 \\
 & \underline{\overset{-}{x^5} \quad\quad \overset{-}{+ x^2}} \\
 & x^4 + x^3 \quad + x + 1 \\
 & \underline{\overset{-}{x^4} \quad\quad \overset{-}{+ x}} \\
 & x^3 \quad + 1 \\
 & \underline{\overset{-}{x^3} \quad \overset{-}{+ 1}} \\
 & \times
\end{array}
$$

Hence, $(x^5 + x^4 + x^3 + x^2 + x + 1) \div (x^3 + 1) = x^2 + x + 1$

(f) $x^3 - 1$ by $x - 1$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
x - 1 & x^3 - 1 \\
 & \underline{\overset{-}{x^3} \overset{+}{- x^2}} \\
 & x^2 - 1 \\
 & \underline{\overset{-}{x^2} \overset{+}{- x}} \\
 & x - 1 \\
 & \underline{x - 1} \\
 & \times
\end{array}
$$

Hence, $(x^3 - 1) \div (x - 1) = x^2 + x + 1$

(g) $3x^3 - x^2 + 4x + 4$ by $3x + 2$

$$
\begin{array}{r|l}
3x + 2 & 3x^3 - x^2 + 4x + 4 \;(\; x^2 - x + 2 \\
 & \underline{\overset{-}{3x^3} \overset{-}{+ 2x^2}} \\
 & -3x^2 + 4x \\
 & \underline{\overset{+}{-3x^2} \overset{+}{- 2x}} \\
 & 6x + 4 \\
 & \underline{6x + 4} \\
 & \times
\end{array}
$$

Hence, $(3x^3 - x^2 + 4x + 4) \div (3x + 2) = x^2 - x + 2$

(h) $p^5 - p^4 + 3p^3 + 4p^2 + 6p + 2$ by $p^2 + 1$

$$p^2 + 1 \,\big)\, p^5 - p^4 + 3p^3 + 4p^2 + 6p + 2 \,\big(\, p^3 - p^2 + 2p + 5$$

$$\begin{array}{l}
p^5 \qquad + p^3 \\
- \qquad\quad - \\
\hline
-p^4 + 2p^3 + 4p^2 \\
-p^4 \qquad\quad -p^2 \\
\hline
2p^3 + 5p^2 + 6p \\
2p^3 \qquad\quad +2p \\
- \qquad\qquad - \\
\hline
5p^2 + 4p + 2 \\
5p^2 + \qquad +5 \\
- \qquad\qquad - \\
\hline
4p - 3
\end{array}$$

Hence, Quotient $= p^3 - p^2 + 2p + 5$

And, Remainder $= 4p - 3$

(i) $3x^4 - 3x^3 + 5x^2 + x - 2$ by $3x^2 - 1$

$$3x^2 - 1 \,\big)\, 3x^4 - 3x^3 + 5x^2 + x - 2 \,\big(\, x^2 - x + 2$$

$$\begin{array}{l}
3x^4 \qquad -x^2 \\
- \qquad\quad + \\
\hline
-3x^3 + 6x^2 + x - 2 \\
-3x^2 \qquad\quad +x \\
\hline
6x^2 - 2 \\
6x^2 - 2 \\
\hline
\times \\
\hline
\end{array}$$

Hence, $(3x^4 - 3x^3 + 5x^2 + x - 2) \div (3x^2 - 1) = x^2 - x + 2$

(j) $x^4 + 10x^3 + 41x - 17$ by $x^2 + 4$

$$x^2 + 4 \,\big)\, x^4 + 10x^3 + 41x - 17 \,\big(\, x^2 + 10x - 4$$

$$\begin{array}{l}
x^4 \qquad +4x^2 \\
- \qquad\quad - \\
\hline
10x^3 - 4x^2 + 41x - 17 \\
10x^3 \quad + 40x \\
- \qquad\quad - \\
\hline
-4x^2 + x - 17 \\
-4x^2 \qquad -16 \\
\hline
x - 1 \\
\hline
\end{array}$$

Hence, Quotient $= x^2 + 10x - 4$

And, Remainder $= x - 1$

(k) $a^4 + a^2 - a - 3$ by $a^2 + 6$

$$a^2 + 6 \,\big)\, a^4 + a^2 - a - 3 \,\big(\, a^2 - 5$$

$$\begin{array}{l}
a^4 + 6a^2 \\
- \quad - \\
\hline
\end{array}$$

$$\begin{array}{l} -5a^2 - a - 3 \\ -5a^2 \quad\quad - 30 \\ \underline{\;+ \quad\quad\quad +\;} \\ \quad\quad -a + 27 \end{array}$$

Hence, Quotient $= a^2 - 5$

And, Remainder $= 27 - a$

(l) $u^3 - 14u^2 + 37u - 26$ by $u^2 - 12u + 3$

$$u^2 - 12u + 3\overline{)u^3 - 14u^2 + 37u - 26}(u - 2$$

$$\begin{array}{l} u^3 - 12u^2 + 3u \\ \underline{- \quad + \quad\quad -} \\ \quad -2u^2 + 34u - 26 \\ \quad -2u^2 + 24u - 6 \\ \underline{\quad + \quad\quad - \quad\quad +} \\ \quad\quad\quad 10u - 20 \end{array}$$

Hence, Quotient $= u - 2$

And, Remainder $= 10u - 20$

4. By dividing $(8x^3 + 36x^2 + 54x + 27)$ by $(2x + 3)$

$$2x + 3\overline{)8x^3 + 36x^2 + 54x + 27}(4x^2 + 12x + 9$$

$$\begin{array}{l} 8x^3 + 12x^2 \\ \underline{- \quad\quad -} \\ \quad 24x^2 + 54x \\ \quad 24x^2 + 36x \\ \underline{\quad - \quad\quad -} \\ \quad\quad\quad 18x + 27 \\ \quad\quad\quad 18x + 27 \\ \underline{\quad\quad\quad - \quad -} \\ \quad\quad\quad\quad \times \end{array}$$

Since, $(2x + 3)$ divides $(8x^3 + 36x^2 + 54x + 27)$ exactly.

So, $(2x + 3)$ is a factor of $(8x^3 + 36x^2 + 54x + 27)$.

5. By dividing $(a^3 - 1)$ by $(a - 1)$.

$$\begin{array}{r} a^2 + a + 1 \\ a - 1\overline{)\; a^3 - 1} \end{array}$$

$$\begin{array}{l} a^3 - a^2 \\ \underline{- \quad +} \\ \quad a^2 - 1 \\ \quad a^2 - a \\ \underline{\quad - \quad +} \\ \quad\quad a - 1 \\ \underline{\quad\quad a - 1} \\ \quad\quad \times \end{array}$$

Since, $(a - 1)$ divides $(a^3 - 1)$ exactly.

So, $(a - 1)$ is a factor of $(a^3 - 1)$.

6. By dividing $(a^4 + a^3 + 8a^2 + a + p)$ by $(a^2 + 1)$.

$$a^2 + 1 \overline{)\, a^4 + a^3 + 8a^2 + a + p\,(}\; a^2 + a + 7$$
$$a^4 + a^2$$
$$- \quad -$$
$$a^3 + 7a^2 + a + p$$
$$a^3 \qquad + a$$
$$- \qquad -$$
$$7a^2 \quad + p$$
$$7a^2 \quad + 7$$
$$- \qquad -$$
$$p - 7$$

Hence, $(a^4 + a^3 + 8a^2 + a + p)$ is divisible by $(a^2 + 1)$.

So, remainder should be zero.

$\therefore \qquad P - 7 = 0$

So, $\qquad P = 7$

7. By dividing $(2y^3 - 14y + k)$ by $(y + 3)$.

$$y + 3 \overline{)\, 2y^3 - 14y + k\,(}\; 2y^2 - 6y + 4$$
$$2y^3 + 6y^2$$
$$- \quad -$$
$$-6y^2 - 14y + k$$
$$-6y^2 - 18y$$
$$+ \quad +$$
$$4y + k$$
$$4y + 12$$
$$- \quad -$$
$$k - 12$$

Since, $(y + 3)$ is a factor of $(2y^3 - 14y + k)$.

So, remainder should be 0.

$\therefore \qquad k - 12 = 0$

So, $\qquad k = 12$

8. By dividing $(x^4 - 6x^3 + 12x^2 - 17x + 5)$ by $(x^2 - 2x + 3)$.

$$x^2 - 4x + 1$$
$$x^2 - 2x + 3 \overline{)\, x^4 - 6x^3 + 12x^2 - 17x + 5\,(}$$
$$x^4 - 2x^3 + 3x^2$$
$$- \quad + \quad -$$
$$-4x^3 + 9x^2 - 17x$$
$$-4x^3 + 8x^2 - 12x$$
$$+ \quad - \quad +$$
$$x^2 - 5x + 5$$
$$x^2 - 2x + 3$$
$$- \quad + \quad -$$
$$-3x + 2$$

We see that $(-3x+2)$ is remainder.
So, we have to subtract $(2-3x)$ from $x^4 - 6x^3 + 12x^2 - 17x + 5$ to make this exactly divisible by $(x^2 - 2x + 3)$.

Exercise 9.5

1. (a) $(2a+3b)(2a+3b) = (2a+3b)^2 = (2a)^2 + (3b)^2 + 2\times 2a \times 3b$

$= 4a^2 + 9b^2 + 12ab$

(b) $(x+3)(x+3) = (x+3)^2 = (x)^2 + (3)^2 + 2\times x \times 3$

$= x^2 + 9 + 6x$

(c) $\left(\frac{7}{9}x+y\right)\left(\frac{7}{9}x+y\right) = \left(\frac{7}{9}x+y\right)^2 = \left(\frac{7}{9}x\right)^2 + (y)^2 + 2\times\frac{7}{9}x\times y$

$= \frac{49x^2}{81} + y^2 + \frac{14xy}{9}$

(d) $\left(\frac{2}{3}x+5\right)\left(\frac{2}{3}x+5\right) = \left(\frac{2}{3}x+5\right)^2 = \left(\frac{2}{3}x\right)^2 + (5)^2 + 2\times\frac{2}{3}x\times 5$

$= \frac{4}{9}x^2 + 25 + \frac{20x}{3}$

2. (a) $(5x-3y)(5x-3y) = (5x-3y)^2 = (5x)^2 + (3y)^2 - 2\times 5x \times 3y$

$= 25x^2 + 9y^2 - 30xy$

(b) $(y-3)(y-3) = (y-3)^2 = (y^2) + (3)^2 - 2\times y\times 3 = y^2 + 9 - 6y$

(c) $\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right) = \left(\frac{3}{4}x-\frac{5}{6}y\right)^2$

$= \left(\frac{3}{4}x\right)^2 + \left(\frac{5}{6}y\right)^2 - 2\times\frac{3}{4}x\times\frac{5}{6}y$

$= \frac{9}{16}x^2 + \frac{25}{36}y^2 - \frac{5xy}{4}$

(d) $(x^2-5)(x^2-5) = (x^2-5)^2$

$= (x^2)^2 + (5)^2 - 2\times x^2\times 5 = x^4 + 25 - 10x^2$

3. (a) $(5x+9)(5x-9) = (5x)^2 - (9)^2 = 25x^2 - 81$

(b) $(x^3+y^2)(x^3-y^2) = (x^3)^2 - (y^2)^2 = x^6 - y^4$

(c) $(x^2y+3z)(x^2y-3z) = (x^2y)^2 - (3z)^2 = x^4y^2 - 9z^2$

(d) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = (x)^2 - \left(\frac{1}{x}\right)^2 = x^2 - \frac{1}{x^2}$

4. (a) 102^2

$= (100+2)^2$

$= (100)^2 + (2)^2 + 2\times 100\times 2$

$= 10000 + 4 + 400$

$= 10404$

(b) 89^2

$= (100-11)^2$

$= (100)^2 + (11)^2 - 2\times 100\times 11$

$= 10000 + 121 - 2200$

$= 7921$

(c) $(311)^2$

$= (300+11)^2$

$= (300)^2 + (11)^2 + 2\times 300\times 11$

(d) 72^2

$= (70+2)^2$

$= (70)^2 + (2)^2 + 2\times 70\times 2$

$= 90000 + 121 + 6600$
$= 96721$

$= ₹\ 4900 + 4 + 280$
$= 5184$

(e) 118^2
$= (120 - 2)^2$
$= (120)^2 + (2)^2 - 2 \times 120 \times 2$
$= 14400 + 4 - 480$
$= 14404 - 480$
$= 13924$

(f) 989^2
$= (1000 - 11)^2$
$= (1000)^2 + (11)^2 - 2 \times 1000 \times 11$
$= 1000000 + 121 - 22000$
$= 1000121 - 22000$
$= 978121$

(g) 13×7
$= (10 + 3) \times (10 - 3)$
$= (10)^2 - (3)^2$
$= 100 - 9 = 91$

(h) 73×67
$= (70 + 3)(70 - 3)$
$= (70)^2 - (3)^2$
$= 4900 - 9 = 4891$

(i) 9.3×8.7
$= (9.0 + 0.3)(9.0 - 0.3)$
$= (9.0)^2 - (0.3)^2$
$= 81.00 - 0.09$
$= 80.91$

(j) 12.5×11.5
$= (12.0 + 0.5)(12.0 - 0.5)$
$= (12.0)^2 - (0.5)^2$
$= 144.00 - 0.25$
$= 143.75$

(k) 105×95
$= (100 + 5)(100 - 5)$
$= (100)^2 - (5)^2$
$= 10000 - 25$
$= 9975$

(l) 153×147
$= (150 + 3)(150 - 3)$
$= (150)^2 - (3)^2$
$= 22500 - 9$
$= 22491$

5. (a) $49x^2 + 126xy + 81y^2 = (7x)^2 + 2 \times 7x \times 9y + (9y)^2$
$= (7x + y)^2$
Putting $(x = 4)$ and $(y = 7)$, we have
$(7x + y)^2 = [7 \times 4 + 7]^2 = [28 + 7]^2$
$= 35^2 \Rightarrow 1225.$

(b) $4x^2 - 12xy + 9y^2 = (2x)^2 - 2 \times 2x \times 3y + (3y)^2$
$= (2x - 3y)^2$
Putting $(x = 4)$ and $(y = 7)$, we have
$(2x - 3y)^2 = [2 \times 4 - 3 \times 7]^2 = [8 - 21]^2$
$= [-13]^2 \Rightarrow 169$

(c) $(x^4 - y^4) = [(x^2)^2 - (y^2)^2]$
$= (x^2 + y^2)(x^2 - y^2)$
Putting $(x = 4)$ and $(y = 7)$, we have
$(x^2 + y^2)(x^2 - y^2) = [(4)^2 + (7)^2] \times [(4)^2 - (7)^2]$
$= [16 + 49] \times [161 - 49] = 65 \times (-33) \Rightarrow -2145$

6. Given, $36x^2 + 60xy + 25y^2$
$= (6x)^2 + 2 \times 6x \times 5y + (5y)^2$
$= (6x + 5y)^2$
Putting $(x = 4)$ and $(y = -7)$, we have
$(6x + 5y)^2 = [6 \times 4 + 5 \times (-7)]^2 = [24 - 35]^2$
$= [-11]^2 \Rightarrow 121$

7. **Given :** $64a^2 - 112ab + 49b^2 = (8a)^2 - 2\times 8a \times 7b + (7b)^2$

$$= (8a - 7b)^2$$

Putting $a = \frac{1}{2}$ and $b = \frac{-3}{2}$, we have

$$= \left(8\times\frac{1}{2} - 7\times\left(-\frac{3}{2}\right)\right)^2$$

$$= \left(4 + \frac{21}{2}\right)^2 = \left(\frac{8+21}{2}\right)^2 = \left(\frac{29}{2}\right)^2 = \frac{841}{4}$$

8. Simplify using identities :

(a) $133\times 133 - 121\times 121$

$= (133)^2 - (121)^2$

$= (133 + 121)(133 - 121)$

$= 254 \times 12 = 3048$

(b) $5.89\times 5.89 - 0.11\times 0.11$

$= (5.89)^2 - (0.11)^2$

$= (5.89 + 0.11)(5.89 - 0.11)$

$= (6.0)\times(5.78) = 34.68$

(c) $\frac{93\times 93 - 5\times 5}{88}$

$= \frac{(93)^2 - (5)^2}{88}$

$= \frac{(93+5)(93-5)}{88}$

$= \frac{98\times 88}{88} = 98$

(d) $\frac{3.29\times 3.29 - 0.17\times 0.17}{3.12}$

$= \frac{(3.29)^2 - (0.17)^2}{3.12}$

$= \frac{(3.29+0.17)(3.29-0.17)}{3.12}$

$= \frac{3.46\times 3.12}{3.12} = 3.46$

9. Since, $x + \frac{1}{x} = 3$

Squaring the both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2\times x\times\frac{1}{x} = 9$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 9 - 2$$

Hence, $x^2 + \frac{1}{x^2} = 7$

10. Since, $\left(x + \frac{1}{x}\right) = 8$

Squaring the both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = 8^2$$

$$x^2+\left(\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}=64$$

$$x^2+\frac{1}{x^2}+2=64$$

$$x^2+\frac{1}{x^2}=64-2$$

$\therefore$ $$x^2+\frac{1}{x^2}=62$$

Now, Since $$x^2+\frac{1}{x^2}=62$$

Squaring on both sides again.

$$\left(x^2+\frac{1}{x^2}\right)^2=(62)^2$$

$$(x^2)^2+\left(\frac{1}{x^2}\right)^2+2\times x^2\times\frac{1}{x^2}=3844$$

$$x^4+\frac{1}{x^4}+2=3844$$

$$x^4+\frac{1}{x^4}=3844-2$$

$\therefore$ $$x^4+\frac{1}{x^4}=3842$$

Hence, $x^2+\frac{1}{x^2}=62$ and $x^4+\frac{1}{x^4}=3842$

11. Since, $\left(x-\frac{1}{x}\right)=2$

Squaring the both sides, we get

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\left(\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}=4$$

$$x^2+\frac{1}{x^2}-2=4$$

$$x^2+\frac{1}{x^2}=4+2$$

Hence, $$x^2+\frac{1}{x^2}=6$$

12. Since, $x-\frac{1}{x}=6$

Squaring the both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = (6)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x} = 36$$

$$x^2 + \frac{1}{x^2} - 2 \quad = 36$$

$$x^2 + \frac{1}{x^2} = 36 + 2$$

$\therefore$ $$x^2 + \frac{1}{x^2} = 38$$

Now, Since $$x^2 + \frac{1}{x^2} = 38$$

Squaring on both sides again

$$\left(x^2 + \frac{1}{x^2}\right) = 38^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 1444$$

$$x^4 + \frac{1}{x^4} + 2 = 1444$$

$$x^4 + \frac{1}{x^4} = 1444 - 2$$

$\therefore$ $$x^4 + \frac{1}{x^4} = 1442$$

Hence, $x^2 + \frac{1}{x^2} = 38$ and $x^4 + \frac{1}{x^4} = 1442$

13. Since, $x + y = 12$ and $xy = 3$ given

Squaring on both sides, we get

$$(x + y)^2 = (12)^2$$

$$x^2 + y^2 + 2xy = 12 \times 12$$

$$x^2 + y^2 + 2xy = 144$$

$$(x^2 + y^2 + 2 \times 3 = 144 \quad \because \; xy = 3 \text{ given})$$

$$x^2 + y^2 + 6 = 144$$

$$x^2 + y^2 = 144 - 6$$

Hence, $$x^2 + y^2 = 138$$

14. Since, $x - y = 9$ and $xy = 2$, Given

Squaring on both sides, we get

$$(x - y)^2 = (9)^2$$

$$(x)^2 + (y)^2 - 2xy = 81$$

$$x^2 + y^2 - 2 \times 2 = 81 \qquad (\because xy = 2)$$

$$x^2 + y^2 - 4 = 81$$

$$x^2 + y^2 = 81 + 4$$

Hence, $x^2 + y^2 = 85$

15. Since, $x + 5y = 9$ and $xy = 4$

Squaring on both sides, we get

$$(x + 5y)^2 = (9)^2$$

$$x^2 + 25y^2 + 10xy = 81$$

$$(x^2 + 25y^2 + 10 \times 4 = 81 \qquad (\because xy = 4 \text{ given})$$

$$x^2 + 25y^2 + 40 = 81$$

$$x^2 + 25y^2 = 81 - 40$$

Hence, $x^2 + 25y^2 = 41$

16. (a) $(x + 2)(x - 2)(x^2 + 4)$

$$= (x^2 - (2)^2)(x^2 + 4)$$

$$= (x^2 - 4)(x^2 + 4)$$

$$= (x^2)^2 - (4)^2 = x^4 - 16$$

(b) $(x + 1)(x - 1)(x^2 + 1)(x^4 + 1)$

$$= [x^2 - (1)^2](x^2 + 1)(x^4 + 1)$$

$$= (x^2 - 1)(x^2 + 1)(x^4 + 1)$$

$$= [(x^2)^2 - (1)^2](x^4 + 1)$$

$$= (x^4 - 1)(x^4 + 1)$$

$$= (x^4)^2 - (1)^2 = (x^8 - 1)$$

Exercise 9.6

1. (a) $81xyz$ and $27xyz^2$

$\because$ $81xyz = 3 \times 3 \times 3 \times 3 \times x \times y \times z$

And, $27xyz^2 = 3 \times 3 \times 3 \times x \times y \times z \times z$

So, $\text{HCF} = 3 \times 3 \times 3 \times x \times y \times z = 27xyz.$

(b) $5xyz, 7x^2y$ and $35xy^2$

$\because$ $5xyz = 5 \times x \times y \times z$

$7x^2y = 7 \times x \times x \times y$

And, $35xy^2 = 5 \times 7 \times x \times y \times y$

So, $\text{HCF} = x \times y = xy.$

(c) a^2b, abc^2, a^3b^2c

$\because$ $a^2b = a \times a \times b$

$abc^2 = a \times b \times c \times c$

And, $a^3b^2c = a \times a \times a \times b \times b \times c$

So, $\text{HCF} = a \times b = ab$

(d) xy, xy^2, x^2y^2 and xyz

$\because$ $xy = x \times y$

$xy^2 = x \times y \times y$

$x^2y^2 = x \times x \times y \times y$

And, $xyz = x \times y \times z$

So, $\text{HCF} = x \times y = xy$

2. (a) $3x + 12$
$= 3(x + 4)$

(b) $x^3 - x^2$
$= x^2 (x - 1)$

(c) $p^5q^4 - p^3q^2 + p^4q^3$
$= p^3q^2(p^2q^2 - 1 + pq)$

(d) $2x + 4x^2$
$= 2x(1 + 2x)$

(e) $\frac{1}{2}at + t^2$
$= \frac{1}{2}t(a + 2t)$

(f) $2xy - 4x + 18$
$= 2(xy - 2x + 9)$

(g) $20x^{15}y^8 - 15x^8y^5$
$= 5x^8y^5(4x^7y^3 - 3)$

(h) $10a^4b^5 - 20a^6b^3 + 15a^5b^4$
$= 5a^4b^3(2b^2 - 4a^2 + 3ab)$

3. (a) $y(x + 2) + 3(x + 2)$
$= (x + 2)(y + 3)$

(b) $x^2(y + 2) + 9(y + 2)$
$= (y + 2)(x^2 + 9)$

(c) $3x(x - 3y) - 1(x - 3y)$
$= (x - 3y)(3x - 1)$

(d) $x^3(2p + r) + x^2(2p + r)$
$= x^2(2p + r)(x + 1)$

(e) $(x + y)(2x + 3) - (x + y)(x + 5)$
$= (x + y)[2x + 3 - x - 5]$
$= (x + y)(x - 2)$

(f) $5(p - q)^2 - 3(p - q)$
$= (p - q)[5(p - q) - 3]$
$= (p - q)(5p - 5q - 3)$

(g) $6(x + 3b) - 4(x + 3b)^2$
$= 2(x + 3b)[3 - 2(x + 3b)]$
$= 2(x + 3b)[3 - 2x - 6b]$

(h) $x^4(a - 2b)^2 + x^2(a - 2b)^3$
$= x^2(a - 2b)^2[x^2 + (a - 2b)]$
$= x^2(a - 2b)^2(x^2 + a - 2b)$

4. (a) $a^2 + bc + ab + ac$
$= a^2 + ab + bc + ac$
$= a(a + b) + c(a + b)$
$= (a + b)(a + c)$

(b) $ax^2 + ay^2 + bx^2 + by^2$
$= x^2(a + b) + y^2(a + b)$
$= (a + b)(x^2 + y^2)$

(c) $(a + b)^3 + (a + b)^2$
$= (a + b)^2[(a + b) + 1]$
$= (a + b)^2(a + b + 1)$

(d) $x^2y - xz^2 - xy + z^2$
$= x^2y - xy - xz^2 + z^2$
$= xy(x - 1) - z^2(x - 1)$
$= (x - 1)(xy - z^2)$

(e) $a^2 - ac + ab - bc$
$= a(a - c) + b(a - c)$
$= (a - c)(a + b)$

(f) $a^3 - b^2 + a - a^2b^2$
$= a^3 + a - a^2b^2 - b^2$
$= a(a^2 + 1) - b^2(a^2 + 1)$
$= (a^2 + 1)(a - b^2)$

(g) $2ap + bp + 2aq + bq$
$= 2ap + 2aq + bp + bq$
$= 2a(p + q) + b(p + q)$
$= (p + q)(2a + b)$

(h) $x^5 - y^3 + x - x^4y^3$
$= x^5 + x - x^4y^3 - y^3$
$= x(x^4 + 1) - y^3(x^4 + 1)$
$= (x^4 + 1)(x - y^3)$

5. (a) $a^2 - 64$
$= a^2 - 8^2$
$= (a + 8)(a - 8)$

(b) $16x^2 - 1$
$= (4x)^2 - (1)^2$
$= (4x + 1)(4x - 1)$

(c) $y^3 - 81y$
$= y(y^2 - 81)$
$= y(y^2 - 9^2)$
$= y(y+9)(y-9)$

(d) $a^2 - b^2 - 2b - 1$
$= a^2 - (b^2 + 2b + 1)$
$= a^2 - (b+1)^2$
$= [a + (b+1)][a - (b+1)]$
$= (a + b + 1)(a - b - 1)$

(e) $x^2 + 2xy + y^2 - 16$
$= x^2 + 2xy + y^2 - 4^2$
$= (x+y)^2 - 4^2$
$= (x + y + 4)(x + y - 4)$

(f) $x^4 - y^4$
$= (x^2)^2 - (y^2)^2$
$= (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x+y)(x-y)$

(g) $(a+b)^2 - (x-y)^2 = [(a+b) + (x-y)][(a+b) - (x-y)]$
$= (a + b + x - y)(a + b - x + y)$

(h) $(5+3p)^2 - 225 = (5+3p)^2 - (15)^2$
$= (5 + 3p + 15)(5 + 3p - 15)$
$= (3p + 20)(3p - 10)$

6. (a) $a^4 - 10a^2b^2 + 25b^4 = (a^2)^2 - 2 \times a^2 \times 5b^2 + (5b^2)^2$
$= (a^2 - 5b^2)^2 = (a^2 - 5b^2)(a^2 - 5b^2)$

(b) $x^2y^2 - 6xyz + 9z^2 = (xy)^2 - 2 \times xy \times 3z + (3z)^2 = (xy - 3z)^2$
$= (xy - 3z)(xy - 3z)$

(c) $4x^2 - 12x + 9 = (2x)^2 - 2 \times 2x \times 3 + (3)^2$
$= (2x - 3)^2 = (2x - 3)(2x - 3)$

(d) $a^2 + \frac{1}{2}a + \frac{1}{16} = a^2 + 2 \times a \times \frac{1}{4} + \left(\frac{1}{4}\right)^2$
$= \left(a + \frac{1}{4}\right)^2 = \left(a + \frac{1}{4}\right)\left(a + \frac{1}{4}\right)$

(e) $9x^2 - 6x + 1 = (3x)^2 - 2 \times 3x \times 1 + (1)^2$
$= (3x - 1)^2 = (3x - 1)(3x - 1)$

(f) $9p^4 - 24p^2q^2 + 16q^4 - 256 = (3p^2)^2 - 2 \times 3p^2 \times 4q^2 + (4q^2)^2 - (16)^2$
$= (3p - 4q)^2 - (16)^2$
$= [(3p - 4q) + 16][(3p - rq) - 16]$
$= (3p - 4q + 16)(3p - 4q - 16)$

(g) $p^2 + 2pq + q^2 - 4 = (p+q)^2 - (2)^2$
$= [(p+q) + 2][(p+q) - 2]$
$= (p + q + 2)(p + q - 2)$

(h) $a^2 + 9b^2 - 6ab - 25x^2 = a^2 + (3b)^2 - 2 \times a \times 3b - (5x)^2$
$= (a - 3b)^2 - (5x)^2$
$= [(a - 3b) + 5x][(a - 3b) - 5x]$
$= (a - 3b + 5x)(a - 3b - 5x)$

7. (a) $x^2 - y^2 + 10yz - 25z^2 = x^2 - [y^2 - 10yz + 25z^2]$
$= x^2 - [y^2 - 2 \times y \times 5z + (5z)^2]$
$= x^2 - (y - 5z)^2$
$= [x + (y - 5z)][x - (y - 5z)]$
$= (x + y - 5z)(x - y + 5z)$

(b) $4a^2 - 12ab + 9b^2 - 16c^2 = (2a-3b)^2 - (4c)^2$
$= [(2a-3b)+4c][(2a-3b)-4c]$
$= (2a-3b+4c)(2a-3b-4c)$

(c) $(a+2b)^2 - 9c^2 = (a+2b)^2 - (3c)^2$
$= (a+2b+3c)(a+2b-3c)$

(d) $64 - x^2 - y^2 - 2xy = 64 - [x^2 + y^2 + 2xy] = (8)^2 - (x+y)^2$
$= (8+x+y)(8-x-y)$
$= [3+(x+y)][8-(x+y)]$

(e) $81 - 4x^2 - 9y^2 + 12xy = 81 - [4x^2 + 9y^2 - 12xy]$
$= (9)^2 - [(2x)^2 + (3y)^2 - 2\times 2x \times 3y]$
$= (9)^2 - (2x-3y)^2$
$= (9+2x-3y)(9-2x+3y)$
$= [9+(2x-3y)][9-(2x-3y)]$

(f) $(a+3)^2 - 10(a+3) + 25 = (a+3)^2 - 2\times(a+3)\times 5 + (5)^2$
$= [(a+3)-5]^2$
$= [a+3-5]^2$
$= (a-2)^2$
$= (a-2)(a-2)$

(g) $4a^4 - b^4 = (2a^2)^2 - (b^2)^2$
$= (2a^2 + b^2)(2a^2 - b^2)$

(h) $a^2 + 9b^2 - 6ab - 25x^2 = (a)^2 + (3b)^2 - 2\times a \times 3b - 25x^2$
$= (a-3b)^2 - (5x)^2$
$= (a-3b+5x)(a-3b-5x)$

Multiple Choice Questions

1. (b) **2.** (c) **3.** (a) **4.** (b)

Brain Teaser

Write 'T' for 'True' and 'F' for 'False' :

1. False **2.** False **3.** True **4.** True **5.** True

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10 Visualizing Solid Shapes

Exercise 10

1. We know that

$(F+V)-E = 2$

(a) $F = 6, \quad E = 12, \quad V = ?$

$\therefore \quad (6+V)-12 = 2$
$V = 2+12-6$
$V = 2+6$

So, $V = 8$

(b) $F = ?, \quad V = 6, \quad E = 9$

$\therefore \quad (F+6)-9 = 2$

$F+6-9 = 2$

$F-3 = 2$

$F = 3+2$

So, $F = 5$

(c) $F = 8, \quad V = 6, \quad E = ?$

$\therefore \quad (8+6)-E = 2$

$14-E = 2$

$E = 14-2$

So, $E = 12$

(d) $F = ?, \quad V = 12, \quad E = 30$

$\therefore \quad (F+12)-30 = 2$

$F+12-30 = 2$

$F-18 = 2$

$F = 2+18$

So, $F = 20$

(e) $F = 5, V = ?, E = 8$

$\therefore \quad (5+V)-8 = 2$

$5+V-8 = 2$

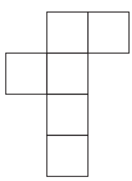
$V = 2+3$

So, $V = 5$

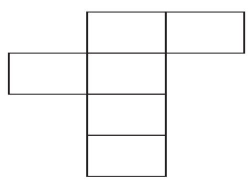
Hence, the missing numbers are :

Faces	6	5	8	20	5
Vertices	8	6	6	12	5
Edges	12	9	12	30	8

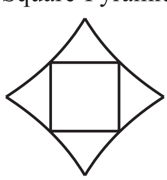
2. (a) Cube



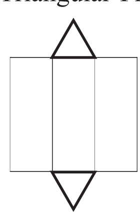
(b) Cuboid



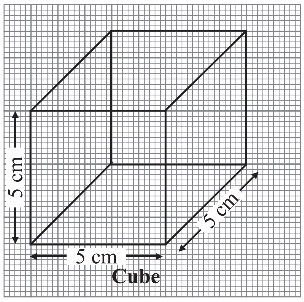
(c) Square Pyramid



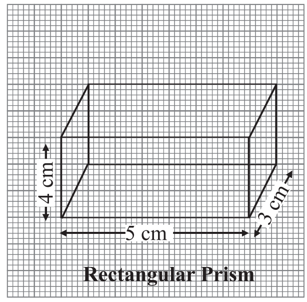
(d) Triangular Prism



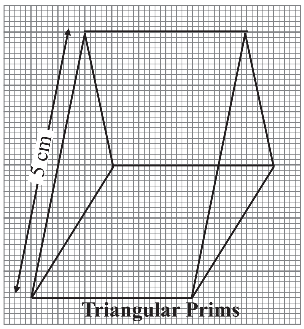
3. (a)



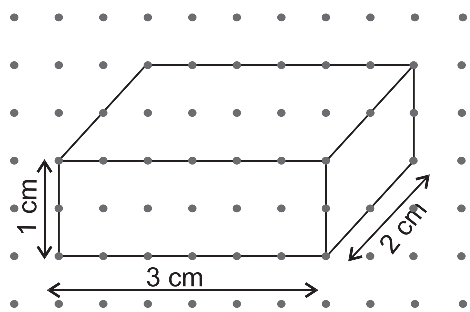
(b)



(c)

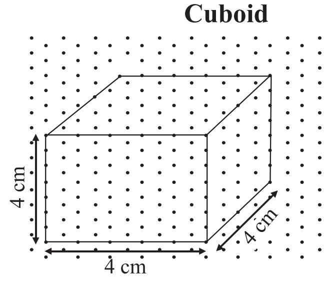


4. (a)



Cuboid

(b)



Cube

Multiple Choice Questions

Tick (✓) the correct option :

1. (a) **2.** (b) **3.** (c) **4.** (d)

HOTS

1. Square prism : *A* square prism has a uniform cross-section which is square. It is also known as a cube.

2. $F = 10, \quad E = 20$ and $\quad V = 15$

By Euler's theorem,

$$V + F = E + 2$$
$$10 + 15 = 20 + 2$$
$$25 \neq 22$$

The given condition does not follow Euler's theorem. So, any polyhedron can not have 10 faces, 20 edges and 15 vertices.

3. It is not possible to have a polyhedron with any given number of faces.

---

11 Mensuration

Exercise 11.1

1. (a) The given, $a = 12$ cm, $b = 35$ cm and $c = 37$ cm

By Hero's formula,

$$S = \frac{a+b+c}{2} = \frac{12+35+37}{2} \text{ cm}$$
$$= \frac{84}{2} \text{ cm} = 42 \text{ cm}$$

$\therefore$ Area of the triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{42(42-12)(42-35)(42-37)} \text{ cm}^2$$
$$= \sqrt{42 \times 30 \times 7 \times 5} \text{ cm}^2$$
$$= \sqrt{7 \times 6 \times 6 \times 5 \times 7 \times 5} \text{ cm}^2$$
$$= 7 \times 6 \times 5 \text{ cm}^2 = 210 \text{ cm}^2.$$

(b) Each side $a = 10$ cm (given)

We know that,

Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} \times (10 \text{ cm})^2$$
$$= \frac{\sqrt{3} \times 100}{4} \text{ cm}^2$$
$$= 25\sqrt{3} \text{ cm}^2.$$

(c) Area of the triangle $= 54$ cm^2

height of triangle $= 9$ cm

base $= ?$

$\because$ Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\therefore$ $54 = \frac{1}{2} \times \text{base} \times 9$

$\text{base} = \frac{54 \times 2}{9}\text{ cm}$

$\text{base} = 6 \times 2\text{ cm} = 12\text{ cm}.$

2. Let the base and height of triangle be $3x$ and $5x$ respectively.

$\because$ The area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\therefore$ $67.5\text{ cm}^2 = \frac{1}{2} \times 3x \times 5x$

$67.5 \times 2\text{ cm}^2 = 15x^2$

$x^2 = \frac{135}{15}\text{ cm}^2$

$x^2 = 9\text{ cm}^2$

$x = \sqrt{9}\text{ cm}$

$x = 3\text{ cm}$

So, the base $= 3x = 3 \times 3\text{ cm} = 9\text{ cm}$

And, the height of triangle $= 5x = 5 \times 3\text{ cm} = 15\text{ cm}.$

3. Each side of a triangle $= 2.4$ cm (given)

$\therefore$ the area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (2.4)^2\text{ cm}^2$

$= \frac{\sqrt{3}}{4} \times 5.76\text{ cm}^2$

$= \sqrt{3} \times 1.44\text{ cm}^2$

$= 1.732 \times 1.44\text{ cm}^2 = 2.494\text{ cm}^2$

4. Length of a rectangle $= 14$ m

Breadth of a rectangle $= 12$ m

Since, the perimeter of the square = the perimeter of a rectangle

$\therefore$ $4 \times \text{side} = 2\,(\text{length} + \text{breadth})$

$2 \times \text{side} = (14 + 12)\text{ m}$

$\text{side} = (26 \div 2)\text{ m}$

side of square $= 13$ m

So, the area of the square $= \text{side}^2 = (13)^2\text{ m}^2 = 169\text{ m}^2$

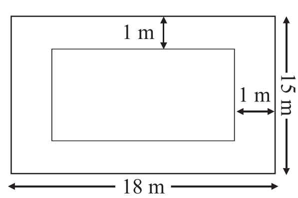
5. Length of floor = 18 m

And, breadth of floor = 15 m

$\therefore$ The area of covering the floor

$= (18-2)(15-2)\text{ m}^2$

$= 16 \times 13\text{ m}^2 = 208\text{ m}^2$



$\because$ Cost of covering the floor = ₹ 105 per m^2

$\therefore$ Total cost of covering the floor $= ₹\ 105 \times \frac{208}{200 \times 100} \times 100 \times 100$

$= ₹\ 105 \times \frac{208}{2}$

$= ₹\ 105 \times 104 = ₹\ 10920$

6. The area of a square park $= 1089\ m^2$

$\therefore$ $side^2 = 1089\ m^2$

$side = \sqrt{1089}\ m$

$side = 33\ m$

Now, the perimeter of the square park $= 4$ side $= 4 \times 33$ m $= 132$ m

$\because$ The cost of fencing the park $= ₹\ 26.5$ per metre

$\therefore$ Total cost of fencing $= ₹\ 26.50 \times 132 = ₹\ 3498$

7. Total cost of a rectangular garden $= ₹\ 1312$

Cost of fencing per meter $= ₹\ 8.00$

So, the perimeter of the rectangular garden $= \frac{1312}{8}$ m $= 164$ m

The perimeter of the rectangular garden $= 2$ (length + breadth)

$164 = 2(46 + \text{breadth})$

$82 = 46 + \text{breadth}$

$\text{breadth} = (82 - 46)\ m = 36\ m.$

8. Length of a rectangular room $= 7.2$ m

Breadth of a rectangular room $= 4$ m

So, the area of the rectangular room $=$ length $\times$ breadth

$= 7.2 \times 4\ m^2 = 28.8\ m^2$

$= 28.8 \times 10000\ cm^2$

$= 288000\ cm^2$

Length of a marble piece $= 60$ cm

Breadth of a marble piece $= 40$ cm

So, area of a marble piece $= l \times b$

$= 60 \times 40\ cm^2 = 2400\ cm^2$

(a) The number of required marble pieces

$= \frac{\text{Area of a rectangular room}}{\text{Area of a marble piece}}$

$= \frac{288000}{2400} = 120$

(b) $\because$ Cost of flooring $= ₹\ 46.50$ per marble piece

$\therefore$ Total cost of flooring $= ₹\ 46.50 \times 120 = ₹\ 5580$

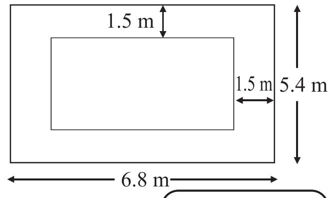
9. Length of a rectangular garden $= 6.8$ m

Breadth of a rectangular garden $= 5.4$ m

$\therefore$ The area of the rectangular garden

$=$ length $\times$ breadth

$= 6.8 \times 5.4\ m^2 = 36.72\ m^2$



Now, the area of the grassy field $= (6.8 - 3) \times (5.4 - 3)\ m^2$
$= 3.8 \times 2.4\ m^2 = 9.12\ m^2$

(a) The area of the flower bed
$=$ The area of the rectangular garden – the area of the grassy field
$= (36.72 - 9.12)\ m^2 = 27.60\ m^2$

(b) $\because$ Cost of growing grass = ₹ 48.50 per m^2
$\therefore$ Total cost = ₹ 43.50 × 9.12 = ₹ 396.72

10. (a) Area of the (horizontal) shaded portion
i.e., $A_1 = 7.5 \times 2\ m^2 = 15\ m^2$
Area of the (vertical) shaded portion
i.e., $A_2 = 5 \times 1.5\ m^2 = 7.5\ m^2$
Area of the dark shaded portion
i.e., $A_3 = 2 \times 1.5\ m^2 = 3\ m^2$
$\therefore$ Area of the shaded portion $= A_1 + A_2 - A_3$
$= (15 + 7.5 - 3)\ m^2 = 19.5\ m^2$

(b) Area of the (horizontal) unshaded portion
i.e., $A_1 = 2 \times 10\ m^2 = 20\ m^2$
Area of the (vertical) unshaded
i.e., $A_2 = 8 \times 2\ m^2 = 16\ m^2$
Area of the cross section
i.e., $A_3 = 2 \times 2\ m = 4\ m^2$
So, area of the shaded portion = Area of outer rectangle
– Area of unshaded portion + area of cross section.
$= 10 \times 8\ m^2 - (A_1 + A_2) + A_3$
$= 80\ m^2 - (20 + 16)\ m^2 + 4\ m^2$
$= 80\ m^2 - 36\ m^2 + 4\ m^2 = 48\ m^2$

11. (a) The perimeter of given figure $= 3 + 9 + 5 + 3 + 8 + 12 = 40$ cm
And, the area of given figure $= 3 \times 9 + 5 \times 3 + 3 \times 3$
$= 27 + 15 + 9 = 51\ cm^2$

(b) The perimeter of given figure $= (6 + 2 + 4 + 2 + 5 + 3 + 15 + 7)$ cm
$= 44$ cm
And, the area of given figure $= 6 \times 7 + 4 \times 5 + 5 \times 3$
$= 42 + 20 + 15 = 77\ cm^2$

(c) The perimeter of the given figure
$= (7 + 1 + 2 + 7 + 2 + 1 + 7 + 1 + 2 + 7 + 2 + 1)$ cm
$= 40$ cm
And, the area of the given figure $= 7 \times 1 + 3 \times 7 + 7 \times 1$
$= 7 + 21 + 7 = 35\ cm^2$

(d) The perimeter of the given figure
$= (8 + 2 + 5 + 3 + 3 + 2 + 3 + 3 + 5 + 2 + 8 + 12)$ m $= 56$ cm
And, the area of the given figure
$= (8 \times 2 + 3 \times 3 + 6 \times 2 + 3 \times 3 + 8 \times 2)\ cm^2$
$= (16 + 9 + 12 + 9 + 16)\ cm^2 = 62\ cm^2$

12. (a) In ΔBCF,

$BF^2 = FC^2 + BC^2$ (Pythagoras theorem)

$7^2 = 4.2^2 + BC^2$

$BC^2 = 49 - 17.64$

$BC^2 = 31.36$

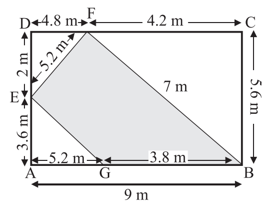
$BC = \sqrt{31.36}$

$BC = 5.6$ m

$AD = BC = 5.6$ m

$AE = AD - ED$

$= (5.6 - 2)$ m $= 3.6$ m $(\because AD = BC)$



Similarly, $DF = DC - FC$

$= (9 - 4.2)$ m $= 4.8$ m $(\because DC = AB)$

So, the area of shaded region

= area of rectangle $ABCD$ – area of ΔBCF – area of ΔDEF – area of ΔAEG

$= 9 \times 5.6 \text{ m}^2 - \frac{1}{2} \times 4.2 \times 5.6 \text{ m}^2 - \frac{1}{2} \times 2 \times 4.8 \text{ m}^2 - \frac{1}{2} \times 3.6 \times 5.2 \text{ m}^2$

$= 50.40 \text{ m}^2 - (11.76 + 4.80 + 9.36) \text{ m}^2 = 50.40 \text{ m}^2 - 25.92 \text{ m}^2$

(b) In ΔAED,

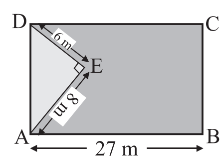
$AD^2 = AE^2 + DE^2$ (Pythagoras theorem)

$AD^2 = 8^2 + 6^2$

$AD^2 = (64 + 36) \text{m}^2$

$AD^2 = 100 \text{ m}^2$

$AD = \sqrt{100}$ m $= 10$ m



So, the area of shaded region

= area of rectangle $ABCD$ – area of ΔAED

$= 27 \times 10 \text{ m}^2 - \frac{1}{2} \times 6 \times 8 \text{ m}^2$

$= 270 \text{ m}^2 - 24 \text{ m}^2 = 246 \text{ m}^2$

Exercise 11.2

1. Area of quadrilateral $ABCD$

= Area of triangle ABC + Area of triangle ADC

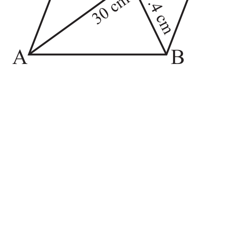
$\therefore$ Area of quadrilateral $= \left(\frac{1}{2} \times \text{Base} \times \text{Height}\right) + \left(\frac{1}{2} \times \times \text{Base} \times \text{Height}\right)$

$= \frac{1}{2} \times 30 \text{cm} + 5.6 \text{cm} + \frac{1}{2} \times 30 \text{cm} \times 7.4 \text{cm}$

$= \frac{1}{2} \times 30 \text{cm} \times [5.6 \text{cm} + 7.4 \text{cm}]$

$= 15 \text{ cm} \times 13 \text{ cm}$

$= 195 \text{ cm}^2$



So, the area of quadrilateral is 195 cm^2.

2. The area of a parallelogram $= 90\text{ m}^2$

Let the base of the parallelogram be x m.

Then, altitude will be $\frac{2}{5}x$ m.

$\because$ Area of parallelogram $=$ base $\times$ height

$$\therefore \quad x \times \frac{2}{5}x = 90\text{ m}^2$$

$$x^2 = \frac{90 \times 5}{2}\text{ m}^2$$

$$x^2 = 225\text{ m}^2$$

$$x = \sqrt{225}\text{ m}$$

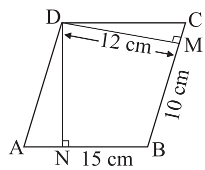
$$x = 15\text{ m}$$

So, base of quadrilateral $= x = 15$ m

And height of quadrilateral $= \frac{2}{5}x = \frac{2}{5} \times 15\text{ m} = 6\text{ m}$

3. We know that,

Area of parallelogram



$$ABCD = \text{base} \times \text{height}$$

$$= BC \times DM$$

$$= 10 \times 12\text{ cm}^2 = 120\text{ cm}^2$$

Also, area of parallelogram $ABCD = 120\text{ cm}^2$

$$AB \times DN = 120\text{ cm}^2$$

$$15\text{ cm} \times DN = 120\text{ cm}^2$$

$$DN = \frac{120}{15}\text{ cm} = 8\text{ cm}$$

Hence, the distance between the longer sides is 8 cm.

4. $d_1 = 12$ cm, $d_2 = 16$ cm

(a) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} \times 12 \times 16\text{ cm}^2 = 96\text{ cm}^2$$

(b) Since $AC = 16$ cm and $BD = 12$ cm

We know that diagonals of a rhombus bisect each other at right angle.

$$\therefore \quad OC = \frac{1}{2}(AC) = \frac{1}{2} \times 16\text{ cm} = 8\text{ cm}$$

$$\text{And,} \quad OB = \frac{1}{2}(BD) = \frac{1}{2} \times 12\text{ cm} = 6\text{ cm}$$

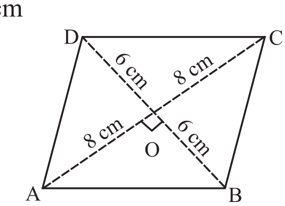
$$\angle BOC = 90°$$

$\therefore$ ΔBOC is a right angled triangle.

By Pythagoras theorem

$$BC^2 = OB^2 + OC^2 = 8^2 + 6^2$$

$$BC^2 = 64 + 36 = 100\text{ cm}^2$$



$\therefore \quad BC = \sqrt{100} = 10$ cm

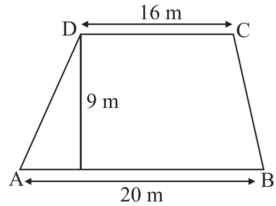
Hence, the length of each side of rhombus is 10 cm.

(c) Perimeter of rhombus $ABCD = 4$ side $= 4 \times 10$ cm $= 40$ cm

(d) Cost of fencing the boundary of the rhombus

$= ₹\ 40 \times 27.5 = ₹\ 1100.$

5. Area of the trapezium



$= \frac{1}{2} \times \text{height} \times \text{sum of parallel sides}$

$= \frac{1}{2} \times 9\text{ m} \times (20 + 16)\text{ m}$

$= \frac{1}{2} \times 9 \times 36\text{ m}^2 = 9 \times 18\text{ m}^2 = 162\text{ m}^2.$

6. Sum of parallel sides $= 60$ cm, $h = ?$

Area of a trapezium $= 660\text{ cm}^2$

$\because$ Area of a trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel side

$\therefore \quad 660 = \frac{1}{2} \times h \times 60$

$660 = 30\,h$

$h = \frac{660}{30} = 22$ cm

Hence, the height of trapezium is 22 cm.

7. The Area of a trapezium $= 448\text{ cm}^2$

height of trapezium $= 14$ cm

Let two parallel sides be $3x$ and $5x$ respectively.

$\because$ The area of a trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel sides

$\therefore \quad 448\text{ cm}^2 = \frac{1}{2} \times 14 \times (3x + 5x)$

$448\text{ cm}^2 = 56x$

$x = \frac{448}{56}$ cm

$x = 8$ cm

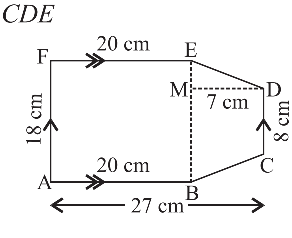
So, the first parallel side $= 3x = 3 \times 8$ cm $= 24$ cm.

And, The second parallel side $= 5x = 5 \times 8$ cm $= 40$ cm.

8. (a) $\therefore$ The area enclosed figure

= Area of □ $ABEF$ + Area of ⏢ $BCDE$

$= (AB \times BE) + \frac{1}{2} \times DM \times (BE + DC)$



$= (20 \times 18)\text{ cm}^2 + \frac{1}{2} \times 7 \times (18 + 8)\text{ cm}^2$

$= 360\text{ cm}^2 + 7 \times 13\text{ cm}^2$

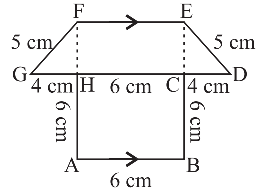
$= 360\text{ cm}^2 + 91\text{ cm}^2 = 451\text{ cm}^2$

(b) Since, $ABCD$ is a square.

So, $AB = HC = 6$ cm $= FE$

In ΔGHF,

$$GF^2 = GH^2 + HF^2 \quad \text{(Pythagoras theorem)}$$
$$(5)^2 = (4)^2 + HF^2$$
$$25 = 16 + HF^2$$
$$HF = \sqrt{25-16} = \sqrt{9}$$
$$HF = 3 \text{ cm}$$



$\therefore$ The area of enclosed figure

$$= \text{Area of } \square\, ABCH + \text{Area of trapezium } GDEF$$
$$= (AB \times BC) + \frac{1}{2} \times HF \times (GD + FE)$$
$$= 6 \times 6 \text{ cm}^2 + \frac{1}{2} \times 3 \times [(4+6+4)+6]$$
$$= 36 \text{ cm}^2 + 10 \times 3 \text{ cm}^2$$
$$= 36 \text{ cm}^2 + 30 \text{ cm}^2 = 66 \text{ cm}^2.$$

9. Area of a trapezium $= 98$ cm^2

height of trapezium $= 7$ cm

Let the second parallel side be x cm

Then, the first parallel side will be $(x+8)$ cm

$\therefore$ Area of a trapezium $= \frac{1}{2} \times$ height $\times$ [sum of parallel sides]

$$98 \text{ cm}^2 = \frac{1}{2} \times 7 \times (x + x + 8)$$
$$\frac{98 \times 2}{7} \text{ cm} = 2x + 8$$
$$2x = (28 - 8) \text{ cm}$$
$$x = \frac{20}{2} = 10 \text{ cm}$$

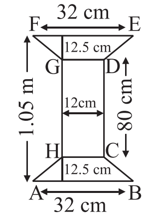
So, the first parallel side $= (x + 8)$ cm

$= (10 + 8)$ cm $= 18$ cm.

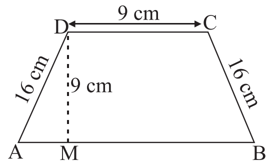
And the second parallel side is $= 10$ cm.

10. $\therefore$ The area of enclosed figure = Area of $ABCH$ + Area of $HCDG$ + Area of $GDEF$

$$= \frac{1}{2} \times HN \times (AB + HC) + HC \times DC + \frac{1}{2} GM \times (GD + FE)$$
$$= \frac{1}{2} \times 12.5 \times (32+12) + 80 \times 12 + \frac{1}{2} \times 12.5 \times (32+12)$$
$$= \frac{1}{2} \times 12.5 \times 44 \text{ cm}^2 + 960 \text{ cm}^2 + \frac{1}{2} \times 12.5 \times 44$$
$$= 275 \text{ cm}^2 + 960 \text{ cm}^2 + 275 \text{ cm}^2$$
$$= 1510 \text{ cm}^2$$



11. The perimeter of a trapezium $= 90$ cm



$\therefore \quad (AB + BC + CD + DA) = 90$ cm

$(AB + CD) + 16 + 16 = 90$ cm

$AB + CD = (90 - 32)$ cm

$AB + CD = 58$ cm

Now, the area of the trapezium $= \frac{1}{2} \times DM \times (AB + CD)$

$= \frac{1}{2} \times 9 \times (58)\ \text{cm}^2$

$= 9 \times 29\ \text{cm}^2 = 261\ \text{cm}^2$

12. Let the second parallel side be x m.

Then, the first parallel side will be $2x$ cm.

$\therefore$ The area of a trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel side

$180\ \text{m}^2 = \frac{1}{2} \times 12 \times (2x + x)$

$180\ \text{m}^2 = 6 \times 3x$

$x = (180 \div 18)$ m

$x = 10$ m

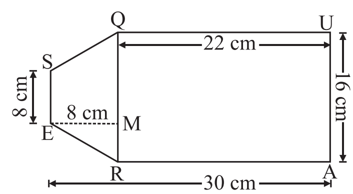
So, the first parallel side is $2x = 2 \times 10 = 20$ m.

And second parallel side is $x = 10$ m.

13. (a) The area of enclosed figure

$=$ Area of ⏢ $SERQ +$ Area of □ $RAUQ$

$= \frac{1}{2} \times EM \times (RQ + ES) + RA \times AU$



$= \frac{1}{2} \times (30 - 22) \times (16 + 8)\ \text{cm}^2$

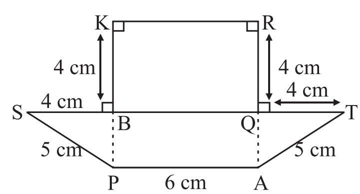
$+ 22 \times 16\ \text{cm}^2$

$= \frac{1}{2} \times 8 \times 24 + 352\ \text{cm}^2$

$= 96\ \text{cm}^2 + 352\ \text{cm}^2 = 448\ \text{cm}^2$

(b) By pythagoras theorem,

In ΔSBP,



$SP^2 = SB^2 + BP^2$

$5^2 = 4^2 + BP^2$

$25 = 16 + BP^2$

$BP^2 = 25 - 16 = 9$

$BP = \sqrt{9}$

$BP = 3$ cm

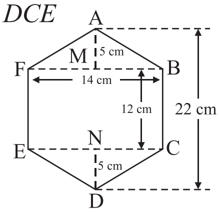
$\therefore$ The area enclosed figure $=$ Area of □ $KBQR +$ ⏢ Area of $SPAT$

$= (KR \times KB) + \frac{1}{2} \times PB \times (ST + PA)$

$$= 6 \times 4 \text{ cm}^2 + \frac{1}{2} \times 3 \times (14 + 6) \text{ cm}^2$$
$$= 24 \text{ cm}^2 + 10 \times 3 \text{ cm}^2$$
$$= 24 \text{ cm}^2 + 30 \text{ cm}^2 = 54 \text{ cm}^2$$

(c) The area of enclosed figure

$= \text{Area of } \Delta AFB + \text{Area of } \square FBCE + \text{Area of } \Delta DCE$



$$= \frac{1}{2} \times FB \times AM + FB \times FE + \frac{1}{2} \times DN \times EC$$
$$= \frac{1}{2} \times 14 \times 5 \text{ cm}^2 + 14 \times 12 \text{ cm}^2 + \frac{1}{2} \times 5 \times 14 \text{ cm}^2$$
$$= 35 \text{ cm}^2 + 168 \text{ cm}^2 + 35 \text{ cm}^2$$
$$= 238 \text{ cm}^2$$

14. The area of the hexagon $TAGORE$ = Area of Δ TBA + Area of ⏢ $ABHG$ + Area of ΔGHO + Area of ΔOSR + Area of ⏢ $RSFE$ + Area of ΔEFT

$$= \frac{1}{2} \times TB \times AB + \frac{1}{2} \times BH \times (AB + GH) + \frac{1}{2} \times HO \times HG + \frac{1}{2} \times OS \times SR$$
$$+ \frac{1}{2} \times FS \times (SR + FE) + \frac{1}{2} \times TF \times FE$$
$$= \frac{1}{2} \times 2 \times 3 \text{ cm}^2 + \frac{1}{2}(TH - TB) \times (3 + 5) \text{ cm}^2 + \frac{1}{2} \times (TO - TH) \times 5 \text{ cm}^2$$
$$+ \frac{1}{2}(TO - TS) \times 2 \text{ cm}^2 + \frac{1}{2} \times (TS - TF) \times (2 + 4) \text{ cm}^2$$
$$+ \frac{1}{2} \times 4 \times 4 \text{ cm}^2$$
$$= 3 \text{ cm}^2 + \frac{1}{2} \times (6 - 2) \times 8 \text{ cm}^2 + \frac{1}{2} \times (9 - 6) \times 5 \text{ cm}^2 + \frac{1}{2}(9 - 7) \times 2 \text{ cm}^2$$
$$+ \frac{1}{2} \times (7 - 4) \times 6 \text{ cm}^2 + 8 \text{ cm}^2$$
$$= 3 \text{ cm}^2 + 16 \text{ cm}^2 + 7.5 \text{ cm}^2 + 2 \text{ cm}^2 + 9 \text{ cm}^2 + 8 \text{ cm}^2$$
$$= 45.5 \text{ cm}^2$$

Exercise 11.3

1. Area of circle $= 1386 \text{ m}^2$ (Given)

$\because$ Area $= \pi r^2$ (Where, r = radius)

$\therefore$ $\pi r^2 = 1386$

$$\frac{22}{7} \times r^2 = 1386 \text{ m}^2$$
$$r^2 = \frac{1386 \times 7}{22} \text{ cm}^2$$
$$r^2 = 63 \times 7$$
$$r^2 = 441 \text{ m}^2$$

$$r = \sqrt{441} \text{ m} = 21 \text{ m}$$

So, radius = 21 m

And, Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 21 \text{ m}^2$

$$= 44 \times 3 \text{ m}^2 = 132 \text{ m}^2$$

2. $\because$ Circumference of circle $= 2\pi r$

$\therefore$ $66 = 2 \times \frac{22}{7} \times r$

$$r = \frac{66 \times 7}{2 \times 22} = 3 \times 3.5 \text{ cm} = 10.5 \text{ cm}$$

So, radius = 10.5 cm

And, Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2$$

$$= 22 \times 1.5 \times 10.5 \text{ cm}^2 = 346.5 \text{ cm}^2$$

3. We know that, $r_1 : r_2 = 3:2$ (Given)

$\because$ Area $= \pi r^2$

So, $A_1 = \pi r_1^2$ ...(i)

$A_2 = \pi r_2^2$ ...(ii)

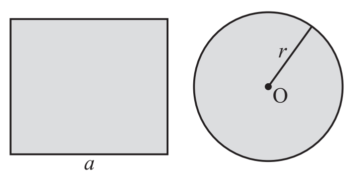
Divide equation (i) by equation (ii), we get

$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

So, the ratio of their areas

i.e., $A_1 : A_2 = 9:4$

4. Given, area of a square $= 121 \text{ m}^2$



Let a be the side of the square

then area $a^2 = 121 \text{ m}^2$

$a = \sqrt{121} \text{ m}$

$a = 11 \text{ m}$

Now, length of the wire = perimeter of the square

$$= 4a = 4 \times 11 \text{ m}$$

$$= 44 \text{ m}$$

Let r be the radius of the circle formed by this wire.

$\therefore$ Circumference of the circle = 44 m

$$2\pi r = 44 \text{ m}$$

$$2 \times \frac{22}{7} \times r = 44 \text{ m}$$

$$r = 7 \text{ m}$$

So, Area of the circle $= \pi r^2 = \frac{22}{7} \times (7)^2 \text{ m}^2$

$$= 22 \times 7 \text{ m}^2 = 154 \text{ m}^2$$

$\therefore$ Area of the region covered increase by $(154 - 121) \text{ m}^2 = 33 \text{ m}^2$

5. Radius of earth $= 6398$ km (Given)

$\therefore$ The length of the equator of the Earth $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 6398 \text{ km}$$
$$= 44 \times 914 \text{ km}$$
$$= 40216 \text{ km}$$

6. Let r_1 be the outer radius of the circle.

Then, r_2 will be the radius of the inner circle.

Therefore, $\pi r_1^2 - \pi r_2^2 = 346.5 \text{ m}^2$

$$(r_1^2 - r_2^2) = \frac{346.5 \times 7}{22} \text{ m}^2$$
$$r_1^2 - r_2^2 = 110.25 \text{ m}^2 \quad \ldots\text{(i)}$$

The circumference of the unshaded region $= 88$ m

$$2\pi r_2 = 88 \text{ m}$$
$$\frac{22}{7} \times r_2 = 44 \text{ m}$$
$$\Rightarrow \quad r_2 = 2 \times 7 = 14 \text{ m}$$

Putting the value of r_2 in equation (i), we get

$$r_1^2 - (14 \text{ m})^2 = 110.25 \text{ m}^2$$
$$r_1^2 - 196 \text{ m}^2 = 110.25 \text{ m}^2$$
$$r_1^2 = 110.25 \text{ m}^2 + 196 \text{ m}^2$$
$$r_1^2 = 306.25 \text{ m}^2$$
$$r_1 = \sqrt{306.25} \text{ m}$$
$$\Rightarrow \quad r_1 = 17.5 \text{ m}$$

So, the circumference of the outer circle

$$= 2\pi r_1 = 2 \times \frac{22}{7} \times 17.5 = \frac{770}{7} \text{ m} = 110 \text{ m}.$$

7. Diameter $= 210$ m (Given)

$\therefore \quad r_1 = 105$ m

$\therefore \quad r_2 = (105 - 5) \text{ m} = 100 \text{ m} \quad (\because r_1 > r_2)$

(a) Area of the path

$= $ Area of the outer circle – Area of the inner circle.

$$= \pi(r_1^2 - r_2^2)$$
$$= \frac{22}{7}(105^2 - 100^2) \text{ m}^2$$
$$= \frac{22}{7}(11025 - 10000) \text{ m}^2$$
$$= \frac{22}{7} \times 1025 \text{ m}^2 = 3221.43 \text{ m}^2$$

(b) Rate of constructing $=$ ₹ 42 per m^2

$\therefore$ Total cost of constructing $=$ ₹ $42 \times \frac{22550}{7} =$ ₹ 135300

8. Diameter of wheel $= 140$ cm (Given)

$\therefore \quad r = \dfrac{140}{2}$ cm $= 70$ cm

Circumference $= 2\pi r = 2 \times \dfrac{22}{7} \times 70$ cm $= 44 \times 10$ cm $= 440$ cm

So, the number of revolutions $= \dfrac{\text{Total covered distance}}{\text{Circumference}}$

$= \dfrac{61.6 \times 100}{440} = 14$

9. Diameter of wheel $= 70$ cm (Given)

$\therefore \quad r = \dfrac{70}{2}$ cm $= 35$ cm

Circumference $= 2\pi r = 2 \times \dfrac{22}{7} \times 35$ cm $= 44 \times 5$ cm $= 220$ cm

Number of revolutions $= 500$

Time $= 5$ min

Total covered distance by a wheel $= 500 \times 220$ cm

$= 110000$ cm or $= 1.1$ km

So, the speed of the wheel $= \dfrac{\text{distance}}{\text{time}} = \dfrac{1.1}{\frac{5}{60}}$ km/h

$= \dfrac{1.1 \times 60}{5}$ km/h $= 13.2$ km/h

10. Let the diameters of two circles be $d_1 d_2$.

So, according to the questions,

$d_1 + d_2 = 280$cm ...(i)

Difference of circumference = 88 cm

$2\pi(r_1 - r_2) = 88$

$2 \times \dfrac{22}{7}(r_1 - r_2) = 88$

$r_1 - r_2 = \dfrac{88 \times 7}{44} = 14$ cm

Hence, the difference of the radii of two circles are 14 cm, 77 cm and 63 cm respectively.

11. (a) We know that,

Area of a semicircle $= \dfrac{1}{2}\pi r^2$

So, the area of the shaded portion $= \dfrac{1}{2}\pi R^2 - 2 \times \dfrac{1}{2}\pi r_s^2$

$= \dfrac{1}{2} \times \dfrac{22}{7} \times \left(\dfrac{AC}{2}\right)^2 - \dfrac{22}{7} \times \left(\dfrac{AB}{2}\right)^2$

$$=\frac{1}{2}\times\frac{22}{7}\times\left(\frac{28}{2}\right)^2-\frac{22}{7}\times\left(\frac{28}{4}\right)^2$$

$$=\frac{22}{7}\times\left(\frac{1}{2}\times196-49\right)\text{cm}^2$$

$$=\frac{22}{7}\times(98-49)\text{ cm}^2=\frac{22}{7}\times49\text{ cm}^2=22\times7\text{ cm}^2=154\text{ cm}^2$$

(b) The area of the shaded portion $=\frac{1}{2}\pi R^2-2\times\frac{1}{2}\pi r^2$

$$=\frac{1}{2}\times\frac{22}{7}\times\left(\frac{21}{2}\right)^2-\frac{22}{7}\times\left(\frac{7}{2}\right)^2$$

$$=\frac{1}{2}\times\frac{22}{7}\times\frac{21\times21}{2\times2}-\frac{22}{7}\times\frac{7\times7}{2\times2}$$

$$=\frac{11\times3\times21}{4}-\frac{11\times7}{2}$$

$$=\left(\frac{693}{4}-\frac{77}{2}\right)\text{cm}^2=\left(\frac{693-154}{4}\right)\text{cm}^2$$

$$=\frac{539}{4}\text{ cm}^2=134.75\text{ cm}^2$$

12. The perimeter of adjoining shaded region

$$\pi OD+\pi BC+\pi AD=\pi[OD+BC+AD]$$

$$=\frac{22}{7}(5+2+3)\text{m}$$

$$=\frac{22}{7}\times10\text{ m}=\frac{220}{7}=31.43\text{ m}$$

The area of adjoining shaded region

$$=\frac{1}{2}\pi OD^2+\frac{1}{2}\pi BC^2-\frac{1}{2}\pi AC^2$$

$$=\frac{\pi}{2}\times(OD^2+BC^2-AC^2)$$

$$=\frac{\pi}{2}\times(OD^2+BC^2-AC^2)$$

$$=\frac{22}{2\times7}(5^2+2^2-3^2)\text{ m}^2$$

$$=\frac{11}{7}\times(25+4-9)\text{ m}^2=\frac{11}{7}\times20\text{ m}^2$$

$$=\frac{220}{7}\text{ m}^2=31.43\text{ m}^2$$

13. The area of the remaining part

= Area of $\square$ $ARKD$ − Area of semi circle

$$= AD \times DK - \frac{1}{2}\pi \times \left(\frac{RA}{2}\right)^2$$

$$= 30 \times 21 \text{ cm}^2 - \frac{1}{2} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \text{ cm}^2$$

$$= 630 \text{ cm}^2 - \frac{11}{7} \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^2$$

$$= 630 \text{ cm}^2 - 173.25 \text{ cm}^2$$

$$= 456.75 \text{ cm}^2.$$

12 Exponents and Powers

Exercise 12.1

1. (a) $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^3$

(b) $\frac{11}{15} \times \frac{11}{15} \times \frac{11}{15} \times \frac{11}{15} = \left(\frac{11}{15}\right)^4$

(c) $\frac{-11}{20} \times \frac{-11}{20} = \left(\frac{-11}{20}\right)^2$

(d) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^5$

2. (a) $\frac{-1}{125} = \frac{-1 \times -1 \times -1}{5 \times 5 \times 5} = \left(\frac{-1}{5}\right)^3$

(b) $\frac{32}{243} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \left(\frac{2}{3}\right)^5$

(c) $\frac{-27}{512} = \frac{-3 \times -3 \times -3}{8 \times 8 \times 8} = \left(\frac{-3}{8}\right)^3$

(d) $\frac{256}{289} = \frac{16 \times 16}{17 \times 17} = \left(\frac{16}{17}\right)^2$

3. (a) $\left(\frac{7}{2}\right)^2 = \frac{7 \times 7}{2 \times 2}$

$= \frac{49}{4}$

(b) $\left(\frac{-4}{5}\right)^4 = \frac{-4 \times -4 \times -4 \times -4}{5 \times 5 \times 5 \times 5}$

$= \frac{256}{625}$

(c) $\left(\frac{-3}{4}\right)^5 = \frac{-3 \times -3 \times -3 \times -3 \times -3}{4 \times 4 \times 4 \times 4 \times 4}$

$= \frac{-243}{1024}$

(d) $\left(\frac{4}{7}\right)^3 = \frac{4 \times 4 \times 4}{7 \times 7 \times 7}$

$= \frac{64}{343}$

(e) $\left(\frac{3}{5}\right)^3 = \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{27}{125}$

(f) $\left(\frac{-1}{3}\right)^7 = \frac{-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1}{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{-1}{2187}$

(g) $\left(\frac{-13}{11}\right)^2 = \frac{-13 \times -13}{11 \times 11} = \frac{169}{121}$

(h) $\left(\frac{-7}{5}\right)^5 = \frac{-7\times-7\times-7\times-7\times-7}{5\times5\times5\times5\times5} = \frac{-16807}{3125}$

4. (a) The reciprocal of $\frac{5}{7} = \frac{7}{5}$

(b) The reciprocal of $9 = \frac{1}{9}$

(c) The reciprocal of $-3 = \frac{-1}{3}$

(d) The reciprocal of $\frac{-11}{3} = \frac{-3}{11}$

(e) The reciprocal of $\left(\frac{1}{13}\right)^4 = (13)^4$

(f) The reciprocal of $\left(\frac{-19}{7}\right)^2 = \left(\frac{7}{-19}\right)^2$

(g) The reciprocal of $\left(\frac{17}{-15}\right)^3 = \left(\frac{-15}{17}\right)^3$

(h) The reciprocal of $\left(\frac{-8}{-5}\right)^5 = \left(\frac{-5}{-8}\right)^5$

5. (a) $\left(\frac{-6}{7}\right)^5 \times \left(\frac{-2}{6}\right)^6 = \frac{-6\times-6\times-6\times-6\times-6}{7\times7\times7\times7\times7}$

$\times \frac{-2\times-2\times-2\times-2\times-2\times-2}{6\times6\times6\times6\times6\times6} = \frac{-32}{50421}$

(b) $\left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 = \frac{2\times2\times2\times2}{3\times3\times3\times3} \times \frac{1\times1}{3\times3} = \frac{16}{729}$

(c) $\left(\frac{-1}{3}\right)^5 \times 2^3 \times \left(\frac{3}{4}\right)^4 = \frac{-1}{3}\times\frac{-1}{3}\times\frac{-1}{3}\times\frac{-1}{3}\times\frac{-1}{3} \times \frac{2}{1}\times\frac{2}{1}\times\frac{2}{1}$

$\times \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{-1}{96}$

(d) $\left(\frac{-4}{7}\right)^2 \times \left(\frac{3}{4}\right)^3 \times \left(\frac{7}{5}\right)^8 = \frac{-4}{7}\times\frac{-4}{7}\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}$

$\times \frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}\times\frac{7}{5}$

$= \frac{3176523}{1562500}$

6. (a) $\left(\frac{3}{11}\right)^8 \div \left(\frac{3}{11}\right)^5 = \left(\frac{3}{11}\right)^{8-5} = \left(\frac{3}{11}\right)^3 = \frac{3\times3\times3}{11\times11\times11} = \frac{27}{1331}$

(b) $\left(\frac{13}{17}\right)^{15} \div \left(\frac{13}{17}\right)^{11} = \left(\frac{13}{17}\right)^{15-11} = \left(\frac{13}{17}\right)^{4}$

$$= \frac{13 \times 13 \times 13 \times 13}{17 \times 17 \times 17 \times 17} = \frac{28561}{83521}$$

(c) $\left(\frac{4}{15}\right)^{6} \div \left(\frac{4}{15}\right)^{9} = \left(\frac{4}{15}\right)^{6-9} = \left(\frac{4}{15}\right)^{-3}$

$$= \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3375}{64}$$

(d) $\left(\frac{15}{19}\right)^{11} \div \left(\frac{15}{19}\right)^{8} = \left(\frac{15}{19}\right)^{11-8} = \left(\frac{15}{19}\right)^{3} = \frac{15 \times 15 \times 15}{19 \times 19 \times 19} = \frac{3375}{6859}$

7. (a) $\left(\frac{-1}{9}\right)^{-1} = \left(\frac{-9}{1}\right)^{1} = -9$ (b) $\left(\frac{7}{13}\right)^{-1} = \left(\frac{13}{7}\right)^{1} = \frac{13}{7}$

(c) $(-5)^{-1} \times \left(\frac{1}{5}\right)^{-1} = \left(\frac{-1}{5}\right)^{1} \times \left(\frac{5}{1}\right)^{1} = \frac{-1}{5} \times \frac{5}{1} = -1$

(d) $(-9)^{-1} = \left(\frac{-1}{9}\right)^{1} = \frac{-1}{9}$

(e) $(6^{-1} - 7^{-1})^{-1} = \left(\frac{1}{6} - \frac{1}{7}\right)^{-1} = \left(\frac{7-6}{42}\right)^{-1} = \left(\frac{1}{42}\right)^{-1} = (42)^{1} = 42$

(f) $\left(\frac{3}{7}\right)^{-1} \times \left(\frac{7}{5}\right)^{-1} = \left(\frac{7}{3}\right)^{1} \times \left(\frac{5}{7}\right)^{1} = \frac{7}{3} \times \frac{5}{7} = \frac{5}{3}$

8. (a) $\left(\frac{9}{5}\right)^{-3} \times \left(\frac{12}{7}\right)^{0} \times 5^{-3} \times \left(\frac{1}{9}\right)^{-1} = \left(\frac{5}{9}\right)^{3} \times 1 \times \left(\frac{1}{5}\right)^{3} \times 9$

$$= \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times 1 \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times 9 = \frac{1}{81}$$

(b) $\left[\left\{\left(\frac{2}{5}\right)^{4}\right\}^{2} \div \left(\frac{2}{5}\right)^{2}\right] \times \left(\frac{1}{2}\right)^{-2} \times 2^{-1} \times \left(\frac{1}{4}\right)^{-1}$

$$= \left[\left\{\frac{2}{5}\right\}^{8} \div \left(\frac{2}{5}\right)^{2}\right] \times \frac{(2)^2}{1^2} \times \frac{1}{2} \times \frac{4}{1}$$

$$= \left[\left(\frac{2}{5}\right)^{8-2}\right] \times \frac{2 \times 2}{1 \times 1} \times \frac{1}{2} \times \frac{4}{1}$$

$$= \left(\frac{2}{5}\right)^{6} \times \frac{2 \times 2 \times 1 \times 4}{1 \times 1 \times 2 \times 1}$$

$$= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \times 4}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 1 \times 1 \times 2 \times 1} = \frac{512}{15625}$$

9. $\left(\frac{2}{5}\right)^{3} \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2a-1} \Rightarrow \left(\frac{2}{5}\right)^{3+(-6)} = \left(\frac{2}{5}\right)^{2a-1}$

$\left(\frac{2}{5}\right)^{3-6} = \left(\frac{2}{5}\right)^{2a-1} \Rightarrow \left(\frac{2}{5}\right)^{-3} = \left(\frac{2}{5}\right)^{2a-1}$

Hence, $2a-1=-3 \Rightarrow 2a=-3+1$

$\Rightarrow 2a=-2$

$a=\frac{-2}{2} \Rightarrow a=-1$

So, the value of a is -1.

10. Let $\left(\frac{-2}{9}\right)^{-3}$ should be divided by x to get $\frac{3}{-8}$.

So, $\left(\frac{-2}{9}\right)^{-3} \div x = \frac{3}{-8}$

$$\left(\frac{9}{-2}\right)^{3} \div x = \frac{3}{-8}$$

$$\left(\frac{9}{-2}\right)^{3} \div \frac{3}{-8} = x$$

$$\frac{9\times 9\times 9}{-2\times -2\times -2} \times \frac{-8}{3} = x$$

$$243 = x$$

or $x = 243$

Hence, $\left(\frac{-2}{9}\right)^{-3}$ should divided by 243 to get $\frac{3}{-8}$

11. (a) $\left(\frac{x}{y}\right)^{-2}$

Given, $\frac{x}{y} = \left(\frac{-2}{3}\right)^{3} \div \left(\frac{3}{5}\right)^{-2} = \left(\frac{-2}{3}\right)^{3} \div \left(\frac{5}{3}\right)^{2}$

$$= \frac{-2\times -2\times -2}{3\times 3\times 3} \times \left(\frac{3}{5}\right)^{2}$$

$$= \frac{-2\times -2\times -2\times 3\times 3}{3\times 3\times 3\times 5\times 5} = \frac{-8}{75}$$

Now, $\left(\frac{x}{y}\right)^{-2} = \left(\frac{-8}{75}\right)^{-2} = \left(\frac{-75}{8}\right)^{2}$

$$= \frac{-75\times -75}{8\times 8} = \frac{5625}{64}$$

(b) $\left(\frac{x}{y}+\frac{y}{x}\right)^{-1}$

Given, $\frac{x}{y}=\frac{-8}{75}$

So, $\left[\frac{-8}{75}+\left(\frac{-75}{8}\right)\right]^{-1}=\left(\frac{-8}{75}-\frac{75}{8}\right)^{-1}=\left(\frac{-64-5625}{600}\right)^{-1}$

$=\left(\frac{-5689}{600}\right)^{-1}=\frac{-600}{5689}$

(c) $\left(\frac{x}{y}\right)^{-\frac{8y}{5x}}$

Given, $\frac{x}{y}=\frac{-8}{75}$

So, $\left(\frac{x}{y}\right)^{\frac{-8y}{5x}}=\left(\frac{-8}{75}\right)^{-\left(\frac{-75\times 8}{8\times 5}\right)}=\left(\frac{-8}{75}\right)^{-(-15)}=\left(\frac{-8}{75}\right)^{15}$

Exercise 12.2

1. (a) $283=2.83\times 10^{2}$ (b) $0.47=4.7\times 10^{-1}$

(c) $28450=2.845\times 10^{4}$ (d) $859731463=8.59731463\times 10^{8}$

(e) $0.00000872=8.72\times 10^{-6}$ (f) $0.000000000007=7.0\times 10^{-12}$

2. (a) $3.4567\times 10^{7}=34567000$ (b) $7.0004\times 10^{7}=70004000$

(c) $1.00075\times 10^{2}=100.075$ (d) $0.9813\times 10^{-5}=0.000009813$

(e) $3.87\times 10^{-3}=0.00387$ (f) $8.0031\times 10^{-3}=0.0080031$

3. (a) $5.3897=5.3897\times 10^{-2}$ (b) $0.00035=3.5\times 10^{3}$

$5.3897=5.3897\times 10^{0}$ $0.00035=3.5\times 10^{-4}$

(c) $0.0001=1.000\times 10^{-5}$ (d) $0.056739=5.6739\times 10^{2}$

$0.0001=1.000\times 10^{-4}$ $0.056739=5.6739\times 10^{-2}$

(e) $3.0012=3.0012\times 10^{-1}$ (f) $30085=30.085\times 10^{-3}$

$3.0012=3.0012\times 10^{0}$ $30085=3.0085\times 10^{4}$

4. The speed of an aircraft is 2.012×10^{3} km/h.

speed = 2.012×10^{3} km/hr

time = 3 hour 30 min = $3\frac{1}{2}$ hour

distance = speed × time

$=2.012\times 10^{3}\times 3\frac{1}{2}$

$=2.012\times 10^{3}\times\frac{7}{2}=7.042\times 10^{3}$ km.

5. speed of light = 300000000 m/sec

or $=3.0\times 10^{8}$ m/s.

6. Mass of proton

$= 0.000,000,000,000,000,000,000,000,000,001673$ kg

$= 1.673 \times 10^{-30}$ kg.

Multiple Choice Questions

1. (b) **2.** (b) **3.** (c) **4.** (b) **5.** (b)

Brain Teaser

1. $\left(\frac{-3}{10}\right)^{-8} \div \left(\frac{-3}{10}\right)^{-6} = \left(\frac{-3}{10}\right)^{-2} = \left(\frac{-10}{3}\right)^{2} = \frac{100}{9}$

2. $\left(\frac{18}{25}\right)^{3} \times \left(\frac{18}{25}\right)^{-5} = \left(\frac{18}{25}\right)^{-2} = \left(\frac{25}{18}\right)^{2} = \frac{625}{324}$

3. $\left[\left(\frac{3}{5}\right)^{0}\right]^{-5} = [1]^{-5} = 1$

4. $\left[\left\{\left(\frac{-8}{21}\right)^{2}\right\}^{3}\right]^{0} = \left[\frac{-8}{21}\right]^{2\times 3\times 0} = \left[\frac{-8}{21}\right]^{0} = 1$

HOTS

1. $a^{x^2-y^2} \times a^{y^2-z^2} \times a^{z^2-x^2} = 1$

L.H.S. $a^{x^2-y^2} \times a^{y^2-z^2} \times a^{z^2-x^2} = a^{x^2-y^2+y^2-z^2+z^2-x^2}$

$a^0 = 1 = \text{R.H.S.}$ **Proved**

2. Mass of Earth $= 5.97 \times 10^{24}$ kg

Mass of Moon $= 7.35 \times 10^{22}$ kg

$\therefore$ Total mass $= 5.97 \times 10^{24}$ kg + 7.35×10^{22} kg

$= 597 \times 10^{22}$ kg + 7.35×10^{22} kg

$= 10^{22}(597 + 7.35)$ kg

$= 604.35 \times 10^{22}$ kg or 6.0435×10^{24} kg.

3. Thickness of a book $= 20$ mm

Thickness of 5 books $= 20 \times 5$ mm $= 100$ mm

Thickness of a paper sheet $= 0.016$ mm

Thickness of 5 paper sheet $= 5 \times 0.016$ mm $= 0.080$ mm

Total thickness of the stack $= 100$ mm + 0.080 mm = 100.080 mm.

13 Direct and Inverse Variations

Exercise 13.1

1. (a)

p	8	16	20	32	60
q	2	4	5	8	15

$$\frac{8}{2}=\frac{4}{1}, \frac{16}{4}=\frac{4}{1}, \frac{20}{5}=\frac{4}{1}, \frac{32}{8}=\frac{4}{1}, \frac{60}{15}=\frac{4}{1}$$

Since, the ratio of the corresponding values of p and q is equal.
So, p is direct variation with q.

(b)

p	5	8	9	11
q	15	24	27	33

$$\frac{5}{15}=\frac{1}{3}, \frac{8}{24}=\frac{1}{3}, \frac{9}{27}=\frac{1}{3}, \frac{11}{33}=\frac{1}{3}$$

Since, the ratio of the corresponding values of p and q is equal.
So, p is direct variation with q.

(c)

p	3	5	6	9	10
q	5	3	10	15	6

$$\frac{3}{5}=\frac{3}{5}, \frac{5}{3}=\frac{5}{3}, \frac{6}{10}=\frac{3}{5}, \frac{9}{15}=\frac{3}{5}, \frac{10}{6}=\frac{5}{3}$$

Since, the ratio of the corresponding values of p and q is not equal p and q do not vary directly.

2. (a) $\frac{60}{4}=\frac{15}{1}$ and $\frac{180}{12}=\frac{15}{1}$

So, $\frac{15}{1}$ is the constant ratio.

Since, x and y vary directly,

$\therefore \quad \frac{x_1}{8}=\frac{15}{1} \quad \Rightarrow \quad x_1=15\times 8=120$

$\frac{x_2}{15}=\frac{15}{1} \quad \Rightarrow \quad x_2=15\times 15=225$

$\frac{x_3}{20}=\frac{15}{1} \quad \Rightarrow \quad x_3=20\times 15=300$

$\frac{x_4}{25}=\frac{15}{1} \quad \Rightarrow \quad x_4=25\times 15=375$

So, the missing entries are :

x	60	**120**	180	**225**	**300**	**375**
y	4	8	12	15	20	25

(b) $\frac{9}{4.5}=\frac{1}{0.5}$

So, $\frac{1}{0.5}$ is the constant ratio.

Since, x and y vary directly

$\therefore \quad \frac{x_1}{3.5} = \frac{1}{0.5} \Rightarrow x_1 = \frac{3.5}{0.5} = 7$

$\frac{x_2}{6.5} = \frac{1}{0.5} \Rightarrow x_1 = \frac{6.5}{0.5} = 13$

$\frac{15}{y_1} = \frac{1}{0.5} \Rightarrow y_1 = 15 \times 0.5 = 7.5$

$\frac{x_3}{9.25} = \frac{1}{0.5} \Rightarrow x_3 = \frac{9.25}{0.5} = 18.50$

$\frac{26.5}{y_2} = \frac{1}{0.5} \Rightarrow y_2 = 13.25$

So, The missing entries are :

x	**7**	9	**13**	15	**18.50**	26.5
y	3.5	4.5	6.5	**7.5**	9.25	**13.25**

3. b, c, and e are direct variation.

4. Let *x* number of notebooks can be bought in ₹ 160 :

No. of note books	15	*x*
Amount (₹)	24	160

$$\frac{x}{15} = \frac{160}{24}$$

$$x = \frac{160 \times 15}{24}$$

$$x = 20 \times 5$$

$$x = 100$$

Hence, he can buy 100 note books for ₹ 160.

5. Let *x* litres of petrol will be needed for a journey of 345 km.

Petrol (in L)	20	*x*
Distance (in km)	115	345

$$\frac{20}{x} = \frac{115}{345}$$

$$x = \frac{20 \times 345}{115}$$

$$x = 60$$

Hence, 60 litres of petrol will be needed for a journey of 345 km.

6. Let she will take *x* minutes to walk 275 m.

Time (in min)	130	*x*
Distance (in m)	110	275

$$\frac{x}{130} = \frac{275}{110}$$

$$x = \frac{275 \times 130}{110}$$

$$x = 325 \text{ minutes}$$

Hence,, Vanshika will take 325 minutes to walk a distance of 275 metres.

7. Let ₹ x will be the worth of 250 us dollars.

Rupees (₹)	7425	x
Dollars ($)	150	250

$$\frac{7425}{x} = \frac{150}{250}$$

$$x = ₹\ \frac{7425 \times 250}{150} = ₹\ 12375$$

Hence, on the same day, ₹ 12375 will be the worth of 250 US dollars.

8. Let, it will cover x km in 25 minutes

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$70 = \frac{\text{Distance}}{1}$$

$$\text{Distance} = 70 \times 1 \text{ km} = 70 \text{ km}$$

Distance (in km)	70	x
Time (in min)	60	25

$$\frac{70}{x} = \frac{60}{25}$$

$$\Rightarrow \quad x = \frac{70 \times 25}{60} \text{ km} = 29.17 \text{ km}$$

Hence, it will cover 29.17 km in 25 minutes.

9. Let the required number of bottles be x :

No. of bottles	5	x
Servings	8	32

So, $$\frac{5}{x} = \frac{8}{32}$$

$$x = \frac{5 \times 32}{8} = 5 \times 4 = 20 \text{ bottles}$$

Hence, the required number of bottles is 20.

10. $5 \text{ men} = 8 \text{ women} \quad \Rightarrow \quad 1 \text{ man} = \frac{8}{5} \text{ women}$

$$\Rightarrow \quad 8 \text{ men} = \frac{8}{5} \times 8 = \frac{64}{5} \text{ women}$$

$\therefore$ 8 men and 12 women $= \frac{64}{5} + 12 = \frac{64+60}{5} = \frac{124}{5}$ women

Let ₹ x be the earning by 8 men and 12 women in a day.

$\therefore$ ₹ x will also be the earning of $\frac{124}{5}$ women in a day.

No. of women	8	$\frac{124}{5}$
Earning (in ₹)	625	x

Note that more the number of women, more will be the earning. So, number of women is in direct variation with the earnings.

$$\therefore \quad \frac{8}{\frac{124}{5}} = \frac{625}{x}$$

$$x = \frac{625 \times 124}{40} = ₹\ 1937.50$$

Hence, 8 men and 12 women earn 1937.50 in one day.

11. Let x be the number of days.

Income (in ₹)	240	1008
Days	5	x

Note that more income, more days

So, number of days is in direct variation with the working days.

$$\frac{240}{1008} = \frac{5}{x}$$

$$x = \frac{5 \times 1008}{240} \text{ days} = 21 \text{ days}$$

Hence, he worked 21 days.

12. Let x g be the required weight.

Weight (in g)	250	x
Length (in cm)	3.5	21

So, $$\frac{250}{x} = \frac{3.5}{21}$$

$\Rightarrow$ $$x = \frac{250 \times 21}{3.5} = 1500 \text{ g}$$

Hence, 1500 g is the required weight of produce an extension of 21 cm.

13. Let x mm be the thickness of 294 sheets of chart paper.

Thickness (in mm)	35	x
No. of sheets	12	294

So, $\frac{35}{x} = \frac{12}{294}$

$x = \frac{35 \times 294}{12}$ mm

$x = 857.5$ mm

Hence, 857.5 mm be the thickness of 294 sheets of chart paper.

14. Let, in x days, it will pick up 7.2×10^8 kg of dust.

Day	12	x
Dust (in kg)	1.8×10^8	7.2×10^8

So, $\frac{12}{x} = \frac{1.8 \times 10^8}{7.2 \times 10^8}$

$x = \frac{12 \times 7.2 \times 10^8}{1.8 \times 10^8}$

$x = 12 \times 4$ days

$x = 48$ days

Hence, the earth it will pick up 7.2×10^8 kg of dust from the atmosphere in 48 days.

15. Let x children can give the performance.

No. of children	35	x
Space (in m^2)	122.5	21

$\frac{35}{x} = \frac{122.5}{21}$

$x = \frac{35 \times 21}{122.5} = 6$ children

Hence, 6 children can give the performance.

Exercise 13.2

1. (a) and (c) are inverse variation.

2. (a) Since, p and q are vary inversely.

So, $pq = \text{constant } (k)$

$k = 9 \times 8 = 72$

Now, $q_1 \times 3 = 72 \Rightarrow q_1 = 72 \div 3 = 24$

$p_1 \times 6 = 72 \Rightarrow p_1 = 72 \div 6 = 12$

$q_2 \times 4 = 72 \Rightarrow q_2 = 72 \div 4 = 18$

$p_2 \times 36 = 72 \Rightarrow p_2 = 72 \div 36 = 2$

$\therefore$ the missing entries are :

p	9	3	**12**	4	**2**
q	8	**24**	6	**18**	36

(b) Since, p and q are vary inversely.

So, $pq = k$ (constant)

$k = 12 \times 10 = 120$

Now, $q_1 \times 3 = 120 \Rightarrow q_1 = 120 \div 3 = 40$

$p_1 \times 20 = 120 \Rightarrow p_1 = 120 \div 20 = 6$

$q_2 \times 15 = 120 \Rightarrow q_2 = 120 \div 15 = 8$

$p_2 \times 5 = 120 \Rightarrow p_2 = 120 \div 5 = 24$

∴ the missing entries are :

p	12	3	**6**	15	**24**
q	10	**40**	20	**8**	5

3. Let x cm^3 be the volume of the gas at pressure 400 mm. We may put the given data in a table as under :

Pressure (in mm)	360	400
Volume (in cm^3)	760	x

Clearly, it is a case of inverse variation

So, $360 : 400 = x : 760$

$$\frac{360}{400} = \frac{x}{760}$$

$$\Rightarrow \quad x = \frac{360 \times 760}{400} \text{ cm}^3$$

$$\Rightarrow \quad x = 9 \times 76 \text{ cm}^3 = 684 \text{ cm}^3$$

Hence, 684 cm^3 is the volume of the gas at pressure 400 mm.

4. Let x cows will graze the same field in 7 days.

No. of cows	35	x
Day	15	7

The ratio of the number of cows = inverse ratio of the number of days.

So, $35 : x = 7 : 15$

$$\Rightarrow \quad \frac{35}{x} = \frac{7}{15} \quad \Rightarrow \quad x = \frac{35 \times 15}{7}$$

$$\Rightarrow \quad x = 5 \times 15 = 75 \text{ cows}$$

Hence, 75 cows will graze the same field in 7 days.

5. Let x workers take 30 days to complete the work.

No. of workers	1800	x
Day	50	30

The ratio of the number of workers = inverse ratio of the number of days.

So, $1800 : x = 30 : 50$

$$\frac{1800}{x} = \frac{30}{50}$$

$\Rightarrow$ $$x = \frac{1800 \times 50}{30}$$

$\Rightarrow$ $$x = 60 \times 50 = 3000$$

So, more workers needed to complete the work $= 3000 - 1800$

$= 1200$ workers.

6. Let x men must be engaged to reap the harvest in 20 days.

No. of men	20	x
Day	25	20

So, the ratio of the number of men = inverse ratio of the number of days.

$$20 : x = 20 : 25$$

$$\frac{20}{x} = \frac{20}{25}$$

$\Rightarrow$ $$x = \frac{20 \times 25}{20} = 25$$

Hence, 25 men must be engaged to reap the harvest in 20 days.

7. Let 56 pumps take x hours to do the same work.

No. of pumps	21	56
Time (hrs)	36	x

So, the ratio of the number of pumps = inverse ratio of the number of hours.

$$21 : 56 = x : 36$$

$$\frac{21}{56} = \frac{x}{36}$$

$\Rightarrow$ $$x = \frac{21 \times 36}{56} \text{ hrs.} = 13\frac{1}{2} \text{ hrs.} = 13 \text{ hrs. } 30 \text{ min.}$$

Hence, 56 pumps can empty the water tank in 13 hours 30 minutes.

8. Let 18 men take x days to complete the same work.

Number of men	12	18
No. of days	6	x

So, the ratio of the number of men = inverse ratio of the number of days

$$12 : 18 = x : 6$$

$$\frac{12}{18} = \frac{x}{6}$$

$\Rightarrow$ $$x = \frac{12 \times 6}{18} = 4 \text{ days}$$

Hence, 18 men take 4 days to complete the same work.

9. Let the ration last in x days :

Day	8	x
Number of girls	25	40

So, the ratio of the number of days = inverse ratio of the number of girls.

$$8 : x = 40 : 25$$

$$\frac{8}{x} = \frac{40}{25}$$

$$x = \frac{25 \times 8}{40} = 5 \text{ days}$$

Hence, the ration last in 5 days.

10. Let reading x hours a day he can finish it in 8 days.

No. of hours	5	x
No. of days	10	8

So, the ratio of the number of hours = inverse ratio of the number of days.

$$5 : x = 8 : 10$$

$$\frac{5}{x} = \frac{8}{10}$$

$$x = \frac{5 \times 10}{8} \text{ hrs} = 6\frac{1}{4} \text{ hrs} = 6 \text{ hrs } 15 \text{ min}$$

Hence, Reading 6 hrs 15 min a day, Dinesh can finish the book in 8 days.

11. Let 525 people have the same stock of food for x days :

No. of people	350	525
No. of days	15	x

So, the ratio of the number of people = inverse ratio of the number of days.

$$350 : 525 = x : 15$$

$$\frac{350}{525} = \frac{x}{15}$$

$$\Rightarrow \quad x = \frac{350 \times 15}{525}$$

$$x = 10 \text{ days}$$

Hence, 525 people have the same stock of food for 10 days.

12. Let the number of died cows be x.

No. of cows	24	$(24 - x)$
No. of days	10	15

So, the ratio of the number of cows = inverse ratio of the number of days

$$24 : (24 - x) = 15 : 10$$

$$\frac{24}{(24-x)} = \frac{15}{10}$$

$$24 \times 10 = 24 \times 15 - 15x$$

$$15x = 360 - 240$$

$$15x = 120$$

$$x = 8 \text{ cows}$$

Hence, the number of died cows is 8.

13. Let working x hours a day he can finish the book in 10 days.

No. of hours	8	x
No. of days	15	10

Clearly, lesser the number of days to finish a book, the greater number of hours required. So, it is a case of inverse variation we know that,

Ratio of no. of hours = inverse ratio of number of days

$$8 : x = 10 : 15$$

$$\frac{8}{x} = \frac{10}{15}$$

$$\Rightarrow \quad x = \frac{8 \times 15}{10}$$

$$\Rightarrow \quad x = 12 \text{ hours}$$

Hence, Working 12 hours a day, he can finish the book in 10 days.

14. Let the average speed of Priyanka be x km/h.

Speed (in km/h)	14	x
Time (in min)	20	10

So, the ratio of the number of speed = inverse ratio of the number of time.

$$14 : x = 10 : 20$$

$$\frac{14}{x} = \frac{10}{20}$$

$$\Rightarrow \quad x = \frac{14 \times 20}{10} \text{ km/h}$$

$$x = 28 \text{ km/h}$$

Hence, the average speed of Priyanka should be 28 km/h.

15. Let the food is enough for x days.

No. of men	420	(420-70)
No. of days	$(35-10)$	x

Clearly, the more number of men, the number of days will be less for which the provision will last.

So, it is a case of inverse variation.

The ratio of the number of men = inverse ratio of the number of days.

So, $$\frac{420}{420-70}=\frac{x}{35-10}$$

$$x=\frac{420\times 25}{350}=6\times 5=30 \text{ days}$$

Hence, the remaining food is enough for 30 days.

Exercise 13.3

1. Praveen can do a work in 7 days

$\therefore$ Work done by Praveen in 1 day $=\frac{1}{7}$ part

$\therefore$ He will be able to complete in 5 days $=\frac{1}{7}\times 5=\frac{5}{7}$ part of work.

2. 30 baskets were weaved by Jaspreet in 45 days

$\therefore$ 1 basket were weaved by Jaspreet in $\frac{45}{30}$ days.

$\therefore$ She will weave 120 baskets in $\frac{45}{30}\times 120$ days $=45\times 4$ days $=180$ days

3. Suraj can do a work in 4 days

$\therefore$ Suraj's 1 days's work $=\frac{1}{4}$ part

Babar can do the same work in 8 days

$\therefore$ Babar's 1 day's work $=\frac{1}{8}$ part

So, Suraj's + Babar's 1 day's work $=\left(\frac{1}{4}+\frac{1}{8}\right)=\left(\frac{2+1}{8}\right)=\frac{3}{8}$

Hence, Suraj and Babar together can finish the work in $\frac{8}{3}$ days *i.e.*, $2\frac{2}{3}$ days.

4. The time taken by Khalid to finish the work $=5$ days
The time taken by Sameer to finish the work $=10$ days
The time taken by Salman to finish the work $=15$ days

$\therefore$ The work done by Khalid in 1 day $=\frac{1}{5}$ part

$\therefore$ The work done by Sameer in 1 day $=\frac{1}{10}$ part

$\therefore$ The work done by Salman in 1 day $=\frac{1}{15}$ part

The work done by (Khalid + Sammer + Salman) in 1 day

$$=\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{15}\right)=\left(\frac{6+3+2}{30}\right)=\frac{11}{30}$$

Hence, Khaid, Sameer and Salman together can finish the work in $\frac{30}{11}$ days

i.e., $2\frac{8}{11}$ days.

5. The time taken by A and B to finish the work = 18 days

$\therefore \quad (A\text{'s} + B\text{'s})\ 1 \text{ day's work} = \frac{1}{18}$ part

$(A\text{'s} + B\text{'s})\ 8 \text{ day's work} = \frac{8}{18}$ part

The remaining work $= \left(1 - \frac{8}{18}\right) = \frac{10}{18} = \frac{5}{9}$ part

Now, A finish the rest work in 15 days.

$\therefore$ A finish whole work separately$= 15 \times \frac{9}{5}$ days $= 3 \times 9$ days $= 27$ days

A's 1 days work $= \frac{1}{27}$ part

$\therefore \quad B$'s 1 days work $= \left(\frac{1}{18} - \frac{1}{27}\right) = \frac{3-2}{54} = \frac{1}{54}$

Hence, A and B alone can finish the work in 27 days and 54 days respectively.

6. A can do a piece of work in 40 days. And B can do the same work in 45 days.

A's 1 day's work $= \frac{1}{40}$ part

B's 1 day's work $= \frac{1}{45}$ part

$\therefore \quad (A+B)$'s 1 day's work $= \frac{1}{40} + \frac{1}{45} = \left(\frac{9+8}{360}\right) = \frac{17}{360}$

$\therefore \quad (A+B)$'s 10 day's work $= 10 \times \frac{17}{360} = \frac{17}{36}$

The remaining work $= \left(1 - \frac{17}{36}\right) = \left(\frac{36-17}{36}\right) = \frac{19}{36}$

Now, A can do the whole work in 40 days.

$\therefore \quad A$ can do $\frac{19}{36}$ work in $40 \times \frac{19}{36}$ days $= \frac{190}{9}$ days.

Hence, the remaining work is done by A in $21\frac{1}{9}$ days.

7. P can harvest a field in 5 days And Q can harvest the same field in 7 days.

P's 1 days work $= \frac{1}{5}$;

$\therefore \quad P$'s 3 days work $= \frac{3}{5}$

Q's 1 days work $= \frac{1}{7}$

The remaining work of $P = 1 - \frac{3}{5} = \frac{2}{5}$

$(P+Q)$'s 1 days work $= \frac{1}{5} + \frac{1}{7}$

$$= \left(\frac{7+5}{35}\right) = \frac{12}{35}$$

$(P+Q)$'s do 1 work in $\frac{35}{12}$ days

$\therefore$ $(P+Q)$'s do $\frac{2}{5}$ of work in $\frac{35}{12} \times \frac{2}{5} = \frac{7}{6} = 1\frac{1}{6}$ days

Hence the remaining work will be completed in $1\frac{1}{6}$ days.

8. 4 men $= 6$ boys

$\Rightarrow$ 1 man $= \frac{6}{4}$ boys

$\Rightarrow$ 2 men $= \frac{6}{4} \times 2$ boys $= 3$ boys

$\therefore$ 2 men and 4 boys $= 3$ boys + 4 boys $= 7$ boys

6 boys can dig a well in 14 days

$\therefore$ 1 boy can dig a well in 14×6 days

$\therefore$ 7 boys can dig a well in $\frac{14 \times 6}{7}$ days $= 2 \times 6$ days

$= 12$ days

Hence, 2 men and 4 boys will dig, the same well in 12 days.

Multiple Choice Questions

Tick (✓) the correct option :

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (a)

14 Introduction to Graphs

Exercise 14.1

1. Without plotting on the graph, name the quadrant/axis of the following coordinates :

(a) $S(4, 2) \rightarrow$ First quadrant

(b) $T(-8, 4) \rightarrow$ Second quadrant

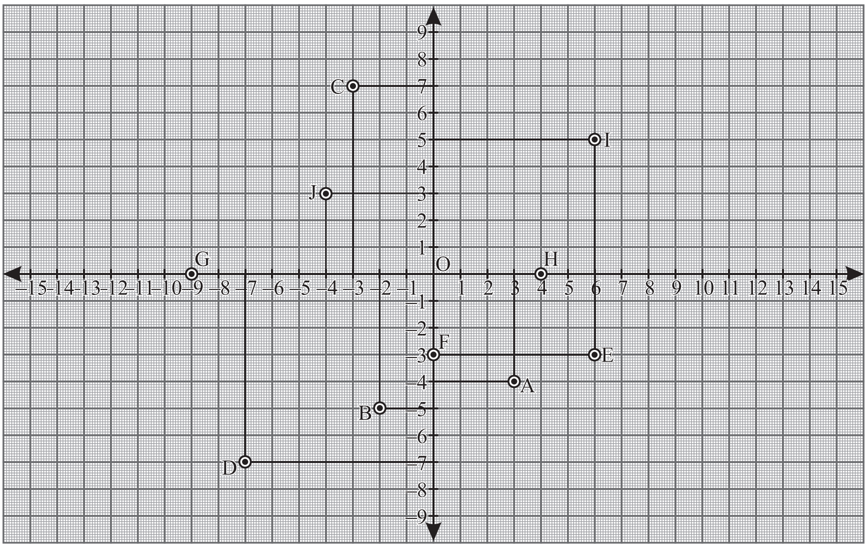
(c) $Z(0, -11) \rightarrow$ y-axis

(d) $R(3, 0) \rightarrow$ x-axis

(e) $L(7, -5) \rightarrow$ Fourth quadrant

(f) $G(-3, -10) \rightarrow$ Third quadrant

2. Plot the following points on graph and name the quadrant/axis :



3. **Find the coordinates of the points *P*, *Q*, *R*, *S*, *T*, *L* and *M* from the following graph :**

P : (−13, 0) *Q* : (5, 5) *R* : (11, – 6)

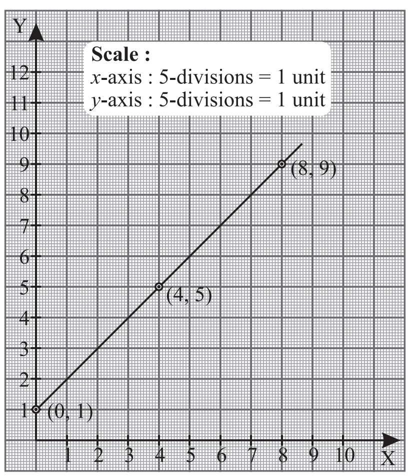
S : (– 11, – 9) *T* : (15, 0) *L* : (1, – 6)

M : (– 7, 3)

Exercise 14.2

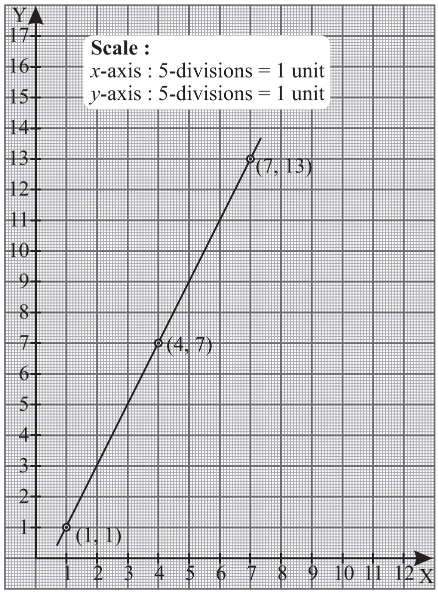
1. (a)

x	0	4	8
$y = x + 1$	**1**	**5**	**9**



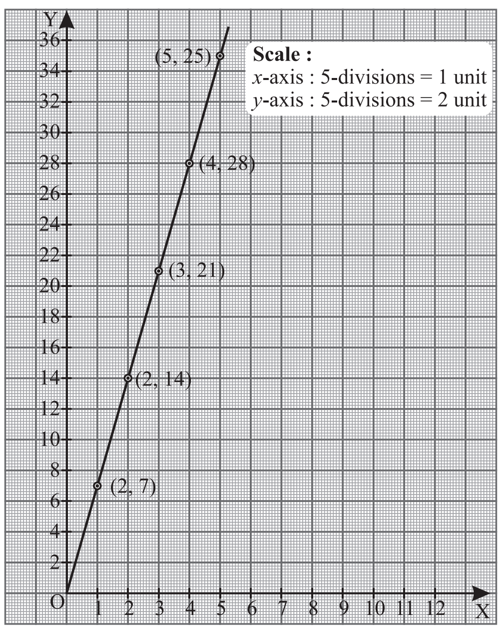
(b)

x	1	4	7
$y = 2x - 1$	1	7	13



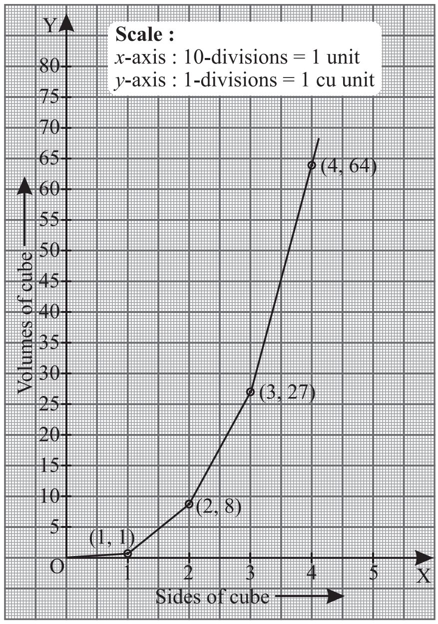
2.

x	1	2	3	4	5
$y = 7x$	7	14	21	28	35



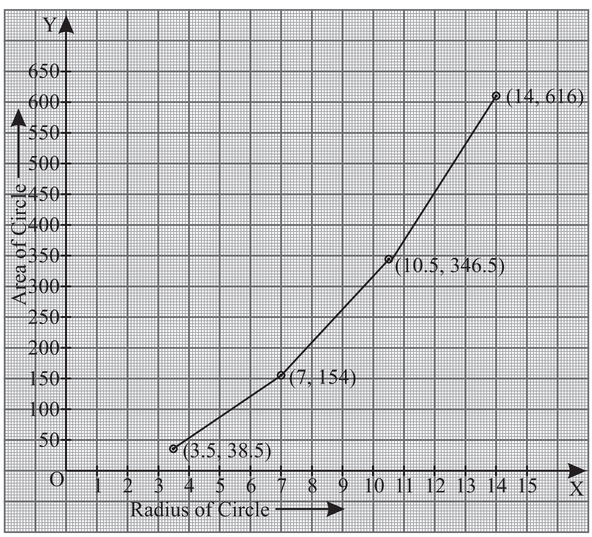
3.

Side of cube (x)	1	2	3	4
Volume of cube ($y = x^3$)	1	8	27	64



4.

Radius of circle	3.5	7.0	10.5	14.0
Area of circle	38.5	154.0	346.5	616.0



5. (a) At the 4th week the plant was no more than 12 cm.

(b) Between 1st and 2nd week there was the greatest increase in height.

(c) The plant grow from 3rd week to 4th week was 2 cm.

6. (a) Distance travelled from C to E is $(80-60)$ m $=20$ metre.

(b) Speed of the car from 20-40 seconds

$$=\frac{\text{Distance}}{\text{Time}}$$

$$=\frac{(60-20)\text{ m}}{(40-20)\text{ sec}}$$

$$=\frac{40\text{ m}}{20\text{ sec}}$$

$$=2\text{ m/sec or 7.2 km/hr.}$$

(c) The speed of the car is zero from 0 to 20 seconds.

7. (a) The speed between 0-4 hrs $=\frac{\text{Distance}}{\text{Time}}$

$$=\frac{420\text{ km}}{4\text{ hr}}$$

$$=105\text{ km/hr.}$$

And the speed between 14-16 hrs $=\frac{\text{Distance}}{\text{Time}}$

$$=\frac{(680-300)\text{ km}}{(16-14)\text{ hr}}$$

$$=\frac{380}{2}\text{ km/hr}$$

$$=190\text{ km/hr.}$$

(b) Speed at 13th hrs is zero km/hr.

(c) Total distance travelled after 16 hrs

= 700 km + 600 km + 200 km + 0 km + 300 km + 180 km

= 1980 km.

8. (a) The number of runs scored in 35 overs is 175 runs.

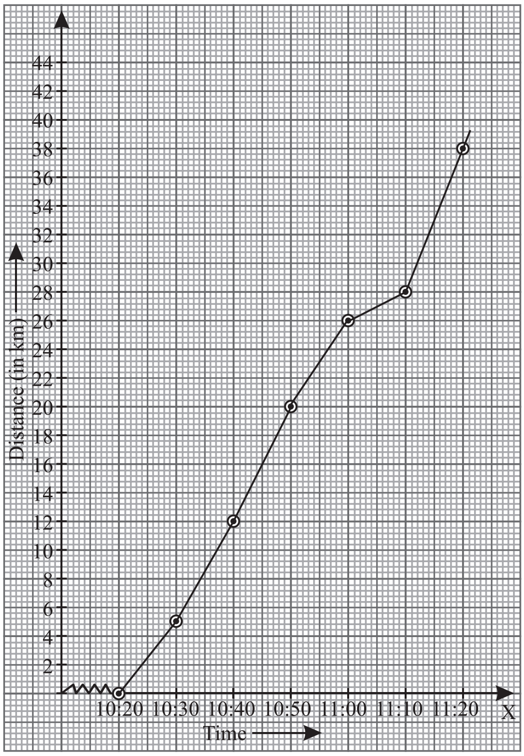
(b) The number of runs scored between 20 to 25 over is (150 – 100) runs = 50 runs.

(c) 25 runs were scored during 0 to 5th overs, 15 to 20th overs and 25 to 35 th overs.

9.

Time	10 : 20 am	10 : 30 am	10 : 40 am	10 : 50 am	11 : 00 am	11 : 10 am	11 : 20 am
Distance (in km)	0	5	12	20	26	28	38

(a)



(b) The car was travelling at the greatest speed between 11 : 10 am to 11 : 20 am.

(c) The speed of the car between 10 : 40 to 10 : 50 am.

$$= \frac{\text{Distance}}{\text{Time}} = \frac{(20-12)\text{ km}}{10\text{ min}}$$

$$= \frac{8}{10} \times 60 \text{ km/hr} \Rightarrow 48 \text{ km/hr.}$$

(d) Total distance trovelled by car = 38 km

And total time taken by car = 6 minutes or 1 hour

So, the average speed of the car $= \frac{\text{Distance}}{\text{Time}} = \frac{38\text{ km}}{1\text{ hr}} = 38\text{ km/hr.}$

Multiple Choice Questions

1. (c), **2.** (c), **3.** (b), **4.** (d), **5.** (b).

Brain Teaser

1. Fill in the blanks :

(a) The signs of a coordinate in 2nd quadrant are **(– ve, + ve)**.

(b) The ordered pair of origin '*O*' is written as **(0, 0)**.

(c) The coordinate $T(-3, -5)$ lies in **3rd** quadrant.

(d) The horizontal axis of a Cartesian plane is called ***X*-axis**.
(e) The coordinate $L(0, -7)$ lies in/on ***Y*-axis**.

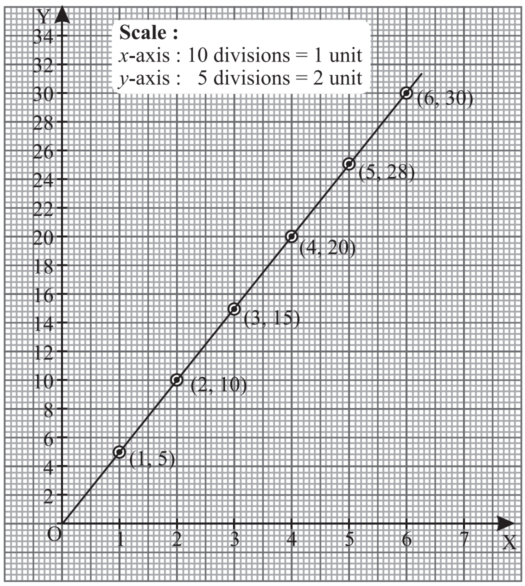
2. Write 'T' for 'True' or 'F for 'False' :
(a) False, (b) True, (c) False, (d) True, (e) False

Hots : Higher Order Thinking Skills

1. A horizontal line $X'OX$ also called X-axis and a vertical line YOY' called Y-axis are divided a pl;ane into 4 parts. Each of these parts is termed as **Quadrants**.

2.

x	1	2	3	4	5	6
$y = 5x$	5	10	15	20	25	30



3. Do it yourself.

15 Playing With Numbers

Exercise 15.1

1. Find the values of the unknowns.

(a)
$$\begin{array}{r} 7\;x \\ -\,y\;3 \\ \hline 3\;9 \end{array}$$

From I column we have $x - 3 = 9 \quad \Rightarrow \quad x = 9 + 3 = 12$

So, x must be $= 12$

But $\qquad x \neq 12$

So x must be only 2 and carrying 1 from left side we have 12 and leaving behind $7 - 1 = 6$

Now, It column $\qquad 6 - y = 3$

$y = 6 - 3$

So, that $\qquad y = 3$

Hence, $x = 2$ and $y = 3$

(b) $6\,xy \times 4 = 2\,x\,72$

$$\begin{array}{cccc} & 6 & x & y \\ & & \times & 4 \\ \hline 2 & x & 7 & 2 \end{array}$$

We have, $4 \times y = 2$

It is possible only when $y = 3$ or $y = 8$

Taking $y = 8$

We have,

$$\begin{array}{cccc} & 6 & x & 8 \\ & & \times & 4 \\ \hline 2 & x & 7 & 2 \end{array}$$

Now, $\qquad x \times 4 + 3 = 7$

It is only possible when $x = 1$ or $x = 6$

But according to ans. x must be greater than 4, so we take $x = 6$.

Now,

$$\begin{array}{cccc} & 6 & 6 & 8 \\ & & \times & 4 \\ \hline 2 & 6 & 7 & 2 \\ \hline \end{array}$$

Taking $y = 3$

We have,

$$\begin{array}{cccc} & 6 & x & 3 \\ & & \times & 4 \\ \hline 2 & x & 7 & 2 \end{array}$$

Now, $4 \times x + 1 = 7$

It is possible only when $x = 4$ or $x = 9$

But according to ans x must be greater than 4 and less than 8.

Hence, $x = 6$ and $y = 8$.

(c) $7xy \div z = zz$

or
$$\begin{array}{ccc} & z & z \\ & \times & z \\ \hline 7 & x & y \\ \hline \end{array}$$

z may be any number from 0 to 9 but it can not be 0, 1, 2, 3, 4, 5, 6, 7 and 9 because no. of digits satisfied this equation only 8 satisfied this sum :

$$\text{So,}\quad \begin{array}{r} z\ z \\ \times\ z \\ \hline 7\ x\ y \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 8\ 8 \\ \times\ 8 \\ \hline 7\ 0\ 4 \\ \hline \end{array}$$

On comparing both sides, we have

Hence, $x = 0,\ y = 4$ and $z = 8.$

(d)

$$\begin{array}{r} 6\ z\ y \\ +8\ 9\ 6 \\ \hline 1\ x\ 3\ 0 \end{array}$$

$$y + 6 = 0$$

It is possible only when $y = 4$

So, that $4 + 6 = 10$ and 1 tens carry to next column.

Now, $1 + z + 9 = 3$

So, that $z = 3$

Now, In next column

$$1 + 6 + 8 = 1x$$

$$15 = 1x$$

On comparing both sides $x = 5$

So, $x = 5,\ y = 4,$ and $z = 3.$

2.

15	20	19
22	18	14
17	16	21

3. (a) $2\ 6 \times 4 = 104$ (b) $1\ 2\ 3 \times 3 = 369$

(c) $2\ 3\ 6 \times 4 = 944$ (d) $0\ 8\ 0 \times 3 = 240$

4. (a) $83 = 8 \times 10 + 3$ (b) $99 = 9 \times 10 + 9$

(c) $373 = 3 \times 100 + 7 \times 10 + 3$

5.

31	32	33	34
35	36	37	38
39	40	41	42
43	44	45	46

6.

46	32	33	43
35	41	40	38
39	37	36	42
34	44	45	31

Magic sum is 154.

Exercise 15.2

1. We know that,
A number is divisible by 2 if its unit digit is 0, 2, 4, 6 or 8.
So, (a) 562 and (c) 950 are divisible by 2.

2. We know that a number is divisible by 3 if the sum of its digits is divisible by 3.
and a number is divisible by 9 if the sum of its digits is divisible by 9.

(a) 93
Sum of the digits $= 9 + 3 = 12$, which is divisible by 3 and not by 9.
So, 93 is divisible by only 3.

(b) 111
Sum of the digits $= 1 + 1 + 1 = 3$, which is is divisible by 3 and not by 9.
So, 111 is divisible by only 3.

(c) 2187
Sum of the digits $= 2 + 1 + 8 + 7 = 18$, which is divisible by 3 and 9 both.
So, 2187 is divisible by 3 and 9 both.

(d) 567
Sum of the digits $= 5 + 6 + 7 = 18$, which is divisible by 3 and 9 both.
So, 567 is divisible by 3 and 9 both

3. A no. is divisible by 10 if its digit at the ones place is 0.
So, (b) 7000, (c) 1250 and (d) 180530 are divisible by 10.

4. Complete the following table :

Number	Divisible by 2	Divisible by 3	Divisible by 5	Divisible by 10
336	3	3	7	7
1200	3	3	3	3
1050	3	3	3	3
7535	7	7	3	7
8242	3	7	7	7

5. We know that a number is divisible by 6 if it is divisible by both 2 and 3.
Since, 21 y 8 has 8 at unit place so it must be divided by 2 and 3 both.
Sum of the digits = $2+1+y+8 = y+11$
Hence, $y+11$ should be multiple of 3, so that y may be 1 or 4 or 7.
Taking $y=1$

We have $= 2118$

Taking $y = 4$ we have $= 2148$

Taking $y = 7$ we have $= 2178$

So, the value of y is 1 or 4 or 7.

6. A number is divisible by 11 if the difference between the sum of its digits at odd places and sum of its digits at even places is either 0 or a multiple of 11.

Sum of digits at odd places $= 1 + 7 + x + 1 = 9 + x$

Sum of digits at even places $= 3 + 2 + 4 + 3 = 12$

For divisiblity of 11

$$9 + x = 12$$
$$x = 12 - 9$$
$$x = 3$$

So, the value of x is 3.

Multiple Choice Questions

1. (b) **2.** (a) **3.** (d) **4.** (a)